THE FUNDAMENTATION OF DECISIONS OF LINEAR PROGRAMMING TYPE THROUGH FUZZY SYSTEMS

1. Preliminaries. Characteristic Function. Fuzzy Sets

Let U be a non-empty set called **univers** or **referential** with its **P**(U) sub-sets called the components of the universe, and a certain sub-set A of referential U, $A \in \mathbf{P}(U)$. Set A is property and uniquely defined by its **characteristic function** f_A with value 1 for every element in A and value 0 for all the other elements of referential U.

$$f_A : U \to \{0,1\}; \quad f_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A. \end{cases} \tag{1}$$

Attempting a better representation of reality, the modern logics also reckons with touches between true (1) and false (0) and accepts the binary classical logics as a particular case. Thus a sentence will enjoy as possible truth value not only one of the extreme values 1 şi 0 (true and false) but also intermediary values specific to each sentence.

A generalization of the concept of classical set may be introduced when dealing with an extension of the co-domain of the characteristic function defined by Ratio (1) from the set with 2 values {0,1} to the entire interval of real numbers [0,1].

This generalization is due to L. Zadeh (1965) [1] and is called **fuzzy set** or **ensemble flou**.

Therefore we shall also consider the set of fuzzy components **F**(U) of referential U. An element A, $A \in \mathbf{F}(U)$ is **fuzzy set** if its characteristic function μ_A has the following form:

$$\mu_A : U \to [0,1] \tag{2}$$

For every element x of the referential $x \in U$ the value of the characteristic function $\mu_A(x)$ is the membership level (or probability) of

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element x to the fuzzy set A. The classical sets are in fact particular cases of fuzzy sets for which the membership levels are but 0 and 1 extreme values.

A **fazzy set** is property defined by the ensemble of the pairs of the form below: $A = \{ (x \mid \mu_A(x)) \}_{x \in U}$ (3)

Example 1: Let there be fuzzy sets A, B on the referential with 5 elements.

$A = \{ x_1 \mid 0; \quad x_2 \mid 1; \quad x_3 \mid 1; \quad x_4 \mid 0; \quad x_5 \mid 1 \}$
$B = \{ x_1 \mid 0{,}2; \quad x_2 \mid 0; \quad x_3 \mid 0{,}3; \quad x_4 \mid 1; \quad x_5 \mid 0{,}8 \}$

As the membership levels for fuzzy set A are but 0 and 1 the set is actually classical $A = \{x_2, x_3, x_5\}$. The elements x_1 and x_4 have a membership level, consequently, they do not belong to classical set A, so $x_1, x_4 \notin A$.

The membership level of element x_1 to fuzzy set B is $\mu_B(x_1) = 0{,}2$. Hence x_1 belongs to B a smoll extent. On the contrary, element x_5 belongs to B to a great extent (0,8) and element x_4 belongs entirely (certainly) to fuzzy set B. The membership level of element x_2 to B is null, which means that it does not (certainty) belong to fuzzy set B.

Both the entire **universe** (referential) U and the classical **empty set** can be considered fuzzy sets properly defined by the following features:

$$\mu_U(x) = 1, \forall x \in U$$
$$\mu_\Phi(x) = 0, \forall x \in U \quad (4)$$

Most of the concepts and properties used with the mathematic operations between classical sets are also met in the theory of fuzzy sets (see also Papers [2] and [3]). One may define the main mathematic operations and relationships between fuzzy sets similarly to the theory of classical sets. In order to do that we should consider any two fuzzy sets on referential U, $A, B \in \mathbf{F}(U)$. The relationships and the new fuzzy sets following from the mathematic operations will be defined by the conditions and/or the definitions of the characteristic due functions:

A=B **equality:** $\mu_A(x) = \mu_B(x), \forall x \in U$

$A \subseteq B$ ($A \subset B$) (strict) inclusion: $\mu_A(x) \le \mu_B(x), \forall x \in U$

$(\exists x' \in U \text{ a.î. } \mu_A(x') \ne \mu_B(x'))$ (5)

$A \cup B$ **(Zadeh) reunion:** $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x)), \forall x \in U$

$A \cap B$ **(Zadeh) intersection:** $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x)), \forall x \in U$

cA complementary: $\mu_{cA}(x) = 1 - \mu_A(x)$, $\forall x \in U$.

Example 2: Let there be fuzzy sets A,B,C in a referential with 5 elements. The next mathematic operations are possible between the sets:

A=	{ x_1 I 0,5;	x_2 I 0;	x_3 I 0,4;	x_4 I 0;	x_5 I 0,7 }
B=	{ x_1 I 0,6;	x_2 I 0,1;	x_3 I 0,3;	x_4 I 0,9;	x_5 I 0,8 }
C=	{ x_1 I 0,2;	x_2 I 0,7;	x_3 I 0,2;	x_4 I 0,8;	x_5 I 0,6 }
$A \cap B$=	{ x_1 I 0,5;	x_2 I 0;	x_3 I 0,3;	x_4 I 0;	x_5 I 0,7 }
$A \cup B$=	{ x_1 I 0,6;	x_2 I 0,1;	x_3 I 0,4;	x_4 I 0,9;	x_5 I 0,8 }
$B \cap A$=	{ x_1 I 0,5;	x_2 I 0;	x_3 I 0,3;	x_4 I 0;	x_5 I 0,7 }
CA=	{ x_1 I 0,5;	x_2 I 1;	x_3 I 0,6;	x_4 I 1;	x_5 I 0,3 }
$A \cap cA$=	{ x_1 I 0,5;	x_2 I 0;	x_3 I 0,4;	x_4 I 0;	x_5 I 0,3 }
$A \cup cA$=	{ x_1 I 0,5;	x_2 I 1;	x_3 I 0,6;	x_4 I 1;	x_5 I 0,7 }

From the example above one infers that $A \cap B = B \cap A$, that is the commutativeness of the crossing operation between fuzzy sets took place. Actually, most of the properties of the operations between classical sets such as: commutativeness, associativeness, distributiveness, idempotency, involution, De Morgan's laws, etc. are also possible for fuzzy sets. But the very important property of univers partition by a fuzzy sub-set as well as its complementary have not been preserved:

$$A \cap cA \neq \Phi \\ A \cup cA \neq U \tag{6}$$

In the theory of fuzzy sets many important results can be proved by using the homologous properties in the classical theory for some associated sets.

Let there be a referential U, a fuzzy set $A \in$ **F**(U), and a non-negative real number $\alpha \in [0,1]$ sub- or unitary. Two very important classical sets associated with fuzzy set A are the **nucleus** and the **support**, respectively:

$$N(A) = \{x \in U \mid \mu_A(x) = 1\}; \quad Sp(A) = \{x \in U \mid \mu_A(x) > 0\} \tag{7}$$

As $1 - \mu_A(x) > 0 \Leftrightarrow \mu_A(x) < 1$ the nucleus of a fuzzy set A together with the support of its complementary form a partition in referential U:

$$N(A) \cap Sp(cA) = \Phi \qquad N(A) \cup Sp(cA) = U \tag{8}$$

Two fuzzy sets associated with fuzzy set A are the A_α **weak cut** and $\overline{A}_\alpha$ **strong cut of level** α (or α-cuts) and their supports:

$$\mu_{A_\alpha}(x)=\begin{cases}\mu_A(x) & ,\mu_A(x)\ge\alpha\\ 0 & ,\mu_A(x)<\alpha\end{cases}\quad \mu_{\bar{A}_\alpha}(x)=\begin{cases}\mu_A(x) & ,\mu_A(x)>\alpha\\ 0 & ,\mu_A(x)\le\alpha\end{cases}\tag{9}$$

$$Sp(A_\alpha)=\{x\in U \mid \mu_A(x)\ge\alpha\}\qquad Sp(\bar{A}_\alpha)=\{x\in U\mid \mu_A(x)>\alpha\}$$

Finally the support is very important that belongs to the strong cut of level 0,5 $Sp(\bar{A}_{0,5})$ a classical set associated with fuzzy set A denoted by $\underline{A}$ and called **associated set:**

$$\underline{A}=\{x\in U\mid \mu_A(x)>0{,}5\}\tag{10}$$

There are some direct properties that belong to the mathematic operations occuring between associated sets, such as:

$$\underline{A\cap B}=\underline{A}\cap\underline{B}\qquad \underline{A\cup B}=\underline{A}\cup\underline{B}\qquad \underline{cA}=c\underline{A}\tag{11}$$

There are many other important properties but because of space reasons we will remind only that the inclusion has been preserved in all types of associated sub-sets:

$$A\subseteq B\Rightarrow\left[\begin{array}{l}\underline{A}\subseteq\underline{B};\ \ N(A)\subseteq N(B);\ \ Sp(A)\subseteq Sp(B);\\ Sp(A_\alpha)\subseteq Sp(B_\alpha);\ \ Sp(\bar{A}_\alpha)\subseteq Sp(\bar{B}_\alpha)\ ,\forall\alpha\in[0,1]\end{array}\right.\tag{12}$$

2. Trapezoidal Fuzzy Numbers

Now let us consider as referential a set of real numbers called **R**. A sub-class of fuzzy sets in **R** is that of the trapezoidal fuzzy numbers **Tp(R)** $\subset$**F(R)**.

A **trapezoidal fuzzy number** $\tilde{a}$ is defined by 4 real numbers; the first is smaller than the second and the last ones are positive:

$$\tilde{a}=(a^m,a^M,a^s,a^d),\quad a^m<a^M;\quad a^s>0;\quad a^d>0\tag{13}$$

The characteristic function of the trapezoidal fuzzy number $\tilde{a}$ is given by the ratio:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x-a^m}{a^s}+1 & ,a^m - a^s < x < a^m \\ 1 & ,a^m \le x \le a^M \\ \dfrac{a^M - x}{a^d}+1 & ,a^M < x < a^M + a^d \\ 0 & ,x \notin (a^m - a^s, a^M + a^d). \end{cases} \tag{14}$$

If the positive numbers a^s and a^d are each other's equal, the trapezoidal fuzzy number is symmetrical, and if $a^m=a^M$, the number is called **fuzzy triangular** (degenerated trapezoidal).

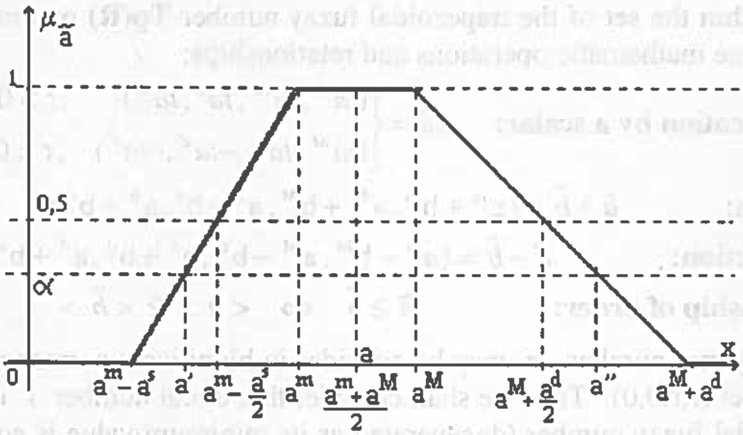

Fig. 1. Graphic Representation of the Characteristic Function of a Trapezoidal Fuzzy Number

The graph of the characteristic function defined in Ratioes (14) and displayed in Figure 1 allow us to interpret the components of a trapezoidal fuzzy number. Thus if one uses a real number **a** for an uncertain affirmation, the number will be consider certain for its entire symmetrical neighbourhood $[a^m,a^M]$. As we are moving away from number a, beyond a^m (the minimum certain value) or a^M (the maximum certain value), the uncertainty level $\mu_{\tilde{a}}$ will linearly decrease. The certainty level ceases to exist (complete uncertainty) beyond a^m-a^s (a^s – left-sided length) and a^M+a^d (a^d – right-sided length), respectively.

Consequently, under uncertainty the real number a is properly represented by the trapezoidal fuzzy number $\tilde{a}$. The 5 classical sets associated with the trapezoidal fuzzy number $\tilde{a}$ and the non-negative real number $\alpha \in [0,1]$ are the following intervals:

$$N(\tilde{a}) = [a^m, a^M] \quad Sp(\tilde{a}) = (a^m - a^s, a^M + a^d)$$
$$\underline{\tilde{a}} = (a^m - \frac{a^s}{2}, a^M + \frac{a^d}{2}) \quad Sp(\tilde{a}_\alpha) = [a', a''] \quad Sp(\overline{\tilde{a}}_\alpha) = (a', a'') \tag{15}$$

With a trapezoidal number the real number below is uniquely associated:

$$< \tilde{a} > = a^m + a^M + \frac{a^d - a^s}{2} \tag{16}$$

Within the set of the trapezoidal fuzzy number **Tp(R)** one may define some some mathematic operations and relationships:

Multiplication by a scalar: $t\tilde{a} = \begin{cases} (ta^m, ta^M, ta^s, ta^d) & , t > 0 \\ (ta^M, ta^m, -ta^d, -ta^s) & , t < 0. \end{cases}$

Addition: $\tilde{a} + \tilde{b} = (a^m + b^m, a^M + b^M, a^s + b^s, a^d + b^d)$ (17)

Substraction: $\tilde{a} - \tilde{b} = (a^m - b^M, a^M - b^m, a^s + b^d, a^d + b^s)$

Relationship of order: $\tilde{a} \geq \tilde{b} \iff < \tilde{a} > \geq < \tilde{b} >$.

Any real number **r** may be consider in biunivocal correspondence to the quartet (r,r,0,0). Thus we shall consider that a real number r is in fact a trapezoidal fuzzy number (degenerated as its minimum value is equal to the maximum one and its both left- and right- sided lenghts are null).

The characteristic due function is as follows:

$$\mu_{\tilde{r}}(x) = \begin{cases} 1 & , x = r \\ 0 & , x \neq r. \end{cases} \tag{18}$$

Now we may say that the addition between the trapezoidal fuzzy numbers is both associative and commutative; it allows $\tilde{0}$ as neutral element but the only interchangeable elements existing are degenerated (real numbers).

$$(\tilde{a} + \tilde{b}) + \tilde{c} = \tilde{a} + (\tilde{b} + \tilde{c}); \; \tilde{a} + \tilde{b} = \tilde{b} + \tilde{a}; \; \tilde{a} + \tilde{0} = \tilde{0} + \tilde{a} = \tilde{a}, \forall \tilde{a}, \tilde{b}, \tilde{c} \tag{19}$$

The multiplication by a scalar (real number) has the following properties:

$$1\tilde{a} = \tilde{a}; \quad t(s\,\tilde{a}) = (ts)\,\tilde{a}; \quad t(\tilde{a} + \tilde{b}) = t\tilde{a} + t\tilde{b}. \tag{20}$$

Example 3. If $\tilde{a}$ =(2, 3, 1, 1) and $\tilde{b}$ =(3, 4, 1, 3) we shall have:

$$<\tilde{a}> = 2+3+\frac{1-1}{2} = 5; \quad <\tilde{b}> = 3+4+\frac{3-1}{2} = 8;$$

$$\tilde{a}+\tilde{b} = (5,7,2,4); \quad \tilde{a}-\tilde{b} = (-2,0,4,2)$$

$$<\tilde{a}+\tilde{b}> = 5+7+\frac{4-2}{2} = 13 = 5+8 = <\tilde{a}> + <\tilde{b}>;$$

$$<\tilde{a}-\tilde{b}> = -2+0+\frac{2-4}{2} = -3 = 5-8 = <\tilde{a}> - <\tilde{b}>.$$

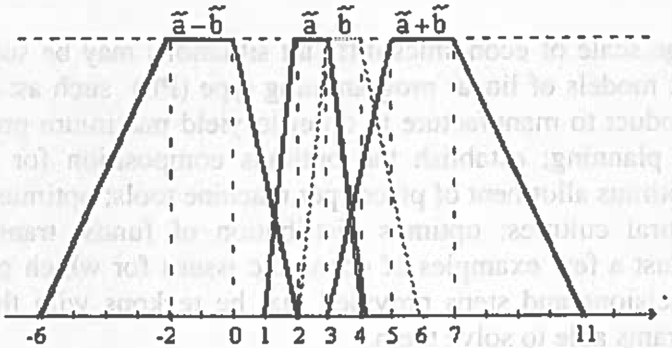

Fig. 2. Graphic Representation of the Addition and Substraction of Trapezoidal Fuzzy Numbers.

The addition, substraction, and multiplication by scalars are met in real numbers associated:

$$<\tilde{a}+\tilde{b}> = <\tilde{a}> + <\tilde{b}>;$$
$$<\tilde{a}-\tilde{b}> = <\tilde{a}> - <\tilde{b}>; \tag{21}$$
$$<t\tilde{a}> = t<\tilde{a}>.$$

as:

$$<\tilde{a}+\tilde{b}> = <(a^m+b^m, a^M+b^M, a^s+b^s, a^d+b^d)> =$$

$$= a^m+b^m+a^M+b^M+\frac{a^d+b^d-a^s-b^s}{2} =$$

$$= a^m+a^M+\frac{a^d-a^s}{2}+(b^m+b^M+\frac{b^d-b^s}{2}) = <\tilde{a}> + <\tilde{b}>;$$

$$<\tilde{a}-\tilde{b}> = <(a^m + b^M, a^M - b^m, a^s + b^d, a^d + b^s)> =$$

$$= a^m - b^M + a^M - b^m + \frac{a^d + b^s - a^s - b^d}{2} =$$

$$= a^m + a^M + \frac{a^d - a^s}{2} - (b^m + b^M + \frac{b^d - b^s}{2}) = <\tilde{a}> - <\tilde{b}>;$$

and

$$<t\tilde{a}> = <(ta^m, ta^M, ta^s, ta^d)> = ta^m + ta^M + \frac{ta^d - ta^s}{2} = t<\tilde{a}>.$$

3. Linear Programs with Trapezoidal Fuzzy Numbers

A large scale of economics difficult situations may be solved through mathematic models of linear programming type (PL) such as: establish the optimus product to manufacture in order to yield maximum profit; optimus production planning; establish the optimus composition for any type of mixture; optimus allotment of pieces per machine tools; optimus distribution of agricultural cultures; optimus distribution of funds; transport matter. These are just a few examples of economic issues for which one may take the best decisions and steps provided that he reckons with the associated linear programs able to solve them.

The level of inflation, the subventions, and other various factors of economic instability do not allow PL coefficients to be accurately established. The study of linear programming is now by far more urgent than ever. It implies to approach fuzzily every coefficient of the linear program in course of study. This is what the paper aims to accomplish by substituting the coefficients for trapezoidal fuzzy numbers.

The general form of a linear programming problem the coefficients of which are trapezoidal fuzzy numbers is given by Ratioes (22):

$$[\max/\min]\,\tilde{z} = \sum_{j=1}^{n} \tilde{c}_j x_j$$

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \substack{\geq \\ \leq} \tilde{b}_i \quad , i = \overline{1, m} \qquad (22)$$

$$x_{1 \ldots n} \geq 0$$

where: the sign $\substack{\geq \\ \leq}$ stands for one of the signs "≥", "=", "≤".

With a linear program in trapezoidal fuzzy numbers the next classical linear program may be associated:

$$[\max/\min] < \tilde{z} > = \sum_{j=1}^{n} < \tilde{c}_j > x_j$$

$$\sum_{j=1}^{n} < \tilde{a}_{ij} > x_j \overset{\geq}{\underset{\leq}{}} < \tilde{b}_i > \quad , i = \overline{1, m} \qquad (23)$$

$$x_{1...n} \geq 0$$

A particularly important result is provided by the theorem below.

Theorem.

The linear program given by Ratioes (22) and (23) are equivalent to one another.

Proof: If X is admissible solution for PL (22), the following series of equivalences takes place:

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \overset{\geq}{\underset{\leq}{}} \tilde{b}_i \Leftrightarrow \sum_{j=1}^{n} (a_{ij}^m, a_{ij}^M, a_{ij}^s, a_{ij}^d) x_j \overset{\geq}{\underset{\leq}{}} (b_i^m, b_i^M, b_i^s, b_i^d) \Leftrightarrow$$

$$\Leftrightarrow \left(\sum_{j=1}^{n} x_j a_{ij}^m, \sum_{j=1}^{n} x_j a_{ij}^M, \sum_{j=1}^{n} x_j a_{ij}^s, \sum_{j=1}^{n} x_j a_{ij}^d\right) \overset{\geq}{\underset{\leq}{}} (b_i^m, b_i^M, b_i^s, b_i^d) \Leftrightarrow$$

$$\Leftrightarrow \sum_{j=1}^{n} \left(a_{ij}^m + a_{ij}^M + \frac{a_{ij}^d - a_{ij}^s}{2}\right) x_j \overset{\geq}{\underset{\leq}{}} \left(b_i^m + b_i^M + \frac{b_i^d - b_i^s}{2}\right) \Leftrightarrow$$

$$\Leftrightarrow \sum_{j=1}^{n} < \tilde{a}_{ij} > x_j \overset{\geq}{\underset{\leq}{}} < \tilde{b}_i >, \forall i = \overline{1, m}$$

So X is also admissible solution for PL (23) and vice versa.

If X^* is the optimus admissible solution for PL (22), the next series of equivalences takes place:

$$\sum_{j=1}^{n} \tilde{c}_j x_j^* \geq / \leq \sum_{j=1}^{n} \tilde{c}_j x_j \Leftrightarrow$$

$$\Leftrightarrow \sum_{j=1}^{n} (c_j^m, c_j^M, c_j^s, c_j^d) x_j^* \geq / \leq \sum_{j=1}^{n} (c_j^m, c_j^M, c_j^s, c_j^d) x_j \Leftrightarrow$$

$$\Leftrightarrow \sum_{j=1}^{n} \left(c_j^m + c_j^M + \frac{c_j^d - c_j^s}{2}\right) x_j^* \geq / \leq \sum_{j=1}^{n} \left(c_j^m + c_j^M + \frac{c_j^d - c_j^s}{2}\right) x_j \Leftrightarrow$$

$$\Leftrightarrow \sum_{j=1}^{n} < \tilde{c}_j > x_j^* \geq / \leq \sum_{j=1}^{n} < \tilde{c}_j > x_j, \forall i \in \overline{1, m}.$$

where: the sign $\geq / \leq$ is replaced by "$\geq$" for [max] problems and "$\leq$" for [min] problems, respectively.

So X^* is also the optimus admissible solution for PL (23) and vice versa.

Linear programs (22) and (23) are equivalent to one another as they have the same sets of both the admissible solutions and the optimus admissible solutions.

The most usual method in solving a PL is **Simplex Algorithm**, elaborated by Dantzig in 1963 [4]. This method is included in many studies (see also [5], [6] and [7]). Primal Simplex Algorithm for PL (22) and PL (23) programs are mutually equivalent as they have homologous criteria for both optimality and base inputs-outputs. The above allow us to indicate the course to follow when dealing with a life situation:

- The coefficients of PL1 linear program generated by the concrete instance are turned into trapezoidal fuzzy numbers by both left- and right- sided distances for the certainty area as well as the lenghts. The distances depend on inflation, subventions, etc. The new linear program in fuzzy numbers will be denoted with PL2.
- With PL2 the equivalent PL (23) linear program called PL3 is associated. It complies with the theorem above.
- We solve the program pair PL2-PL3 through Primal Simlex Algorithm. In the Simplex (numerical) table associated with PL3 the places of the row of the target function coefficients as well as the column of the base coefficients are taken by their homologues from PL2 (trapezoidal fuzzy numbers). The rows $\tilde{z}_j$, $\tilde{c}_j - \tilde{z}_j$ will be also calculated in trapezoidal fuzzy numbers.

When the initial PL1 program has no solutions giving it up is justified no more as the associated PL2-PL3 programs may have optimus solutions. Similarly, if PL1 is non-defined, *i.e.* there is an infinity of solutions. The dilemma of choosing one solution alone may be sometimes overcome in the way just mentioned. It is the way of reaching linear programs of unique optimus solution.

4. Examples of linear programs in trapezoidal fuzzy numbers

Example 4. Production planning.

A certain company uses two kinds of raw materials M_1 and M_2 in order to obtain 2 types T_1 and T_2 of the same product P.

The difference between T_1 and T_2 is due to a quantitative distinction between M_1 and M_2 required by the production process as well as to minor technological adjustments.

A ton of T_1 requires 5 tons M_1 and 4 tons M_2 while a ton of T_2 necessitates 6 tons M_1 and 5 tons M_2. The company enjoys funds for 30 tons M_1 and 20 tons M_2.

A certain locality is intended to be provided with 5,5 tons of product P. A ton of T_1 ensures a benefit of 3 monetary units; the benefit for a ton of T_2 is more important - 7 m.u.

Which are the exact quantities of T_1 and T_2 that yield maximum profit?

PL1, the linear program proper for the case is given by Ratioes (24).

$$[\max]\, z = 3x_1 + 7x_2$$
$$\begin{cases} 5x_1 + 6x_2 \le 30 \\ 4x_1 + 5x_2 \le 20 \quad \text{(PL1)} \\ 2x_1 + 2x_2 \ge 11 \end{cases} \tag{24}$$
$$x_{1,2} \ge 0$$

Usually the subject is dropped as it has no admissible solutions.

Because of the inflation the price of P is constantly raising. Consequently, the benefit per 1 ton P is (probably) bigger than expected. On the other hand, unexpected subventions may occur that keep a lower price than estimated. The above suggest that the unitary benefit of 3 m.u. and 7 m.u., respectively need to be replaced by the trapezoidal fuzzy numbers (2,4,1,3) and (6,8,4,2), respectively. Thus the benefit of 3 m.u. for 1 ton T_1 is considered certain for an area both right- and left- sided called length 1. The certain minimum is 2=3-1 and the certain maximum is 4=3+1. The right- and left- sided lengths going up to an uncertain overal benefit have been considered of 1 m.u. and 3 m.u., respectively.

The decrease in the quantity of row material supply (30 and 20 tons) may be the consequence of inflation (same money buys less) while the increase may be the result of a double action. Exchanging the acquisition funds for hard currency and banking the money until purchasing the supplies is the way of drastically mitigating the inflation effects. Secondly, in many cases there are safe stocks providing for the proper quantity of raw materials.

In the example above the two values (30 and 20) were estimated by means of trapezoidal fuzzy numbers, (29,31,4,2) and (19,21,3,1).

Technology requires raw materials to have certain quality specifics. Should they not comply with the standard, it will be necessary the raw materials quantities involved in the technological process to be either increased or decreased. The example considers 4 technological qualities: 5, 4, 6 şi 5 tons fromed by 4 trapezoidal fuzzy numbers: (4,6,3,1), (3,5,3,1), (5,7,1,3) and (4,6,2,2).

The market requirements (at least 5,5 tons P) are the only data taken here as certain.

Next we display both the linear program in trapezoidal fuzzy numbers in Ratioes (25) and the associated linear program in Ratioes (26):

$$[\max]\,\tilde{z} = (2,4,1,3)x_1 + (6,8,4,2)x_2$$

$$\begin{cases} (4,6,3,1)x_1 + (5,7,1,3)x_2 \le (29,31,4,2) \\ (3,5,3,1)x_1 + (4,6,2,2)x_2 \le (19,21,3,1) \\ (2,2,0,0)x_1 + (2,2,0,0)x_2 \ge (11,11,0,0) \end{cases} \quad \text{(PL2)} \qquad (25)$$

$$x_{1,2} \ge 0$$

$$[\max] < \tilde{z} >= 7x_1 + 13x_2$$

$$\begin{cases} 9x_1 + 13x_2 \le 59 \\ 7x_1 + 10x_2 \le 39 \\ 4x_1 + 4x_2 \ge 22 \end{cases} \quad \text{(PL3)} \qquad (26)$$

$$x_{1,2} \ge 0$$

Primal Simplex Algorithm corresponding to the program pair PL2-PL3 is displayed in Table 1.

The optimus solutions infered from the table are the following:

$$x_1^{\bullet} = \frac{16}{3};\ x_2^{\bullet} = \frac{1}{6};\ (x_3^{\bullet} = \frac{53}{6}).$$

PL2 maximus value for the target function is this:

$$\tilde{z}_{max} = \frac{1}{3}(35, 68, 18, 49) = (11,(6); 22,(6); 6; 16,(3)).$$

The characteristic function is graphically presented in Figure 3.

Table 1.

Primal Simplex Algorithm for Programs (25) and (26).

B	C^B	S^B	<2,4,1,3>	<6,8,4,2>	0	0	M·<-1,-1,0,0>	0
			7	13	0	0	-2M	0
			x_1	x_2	x_3	x_4	x_5	x_6
x_3	0	59	9	13	1	0	0	0
x_4	0	39	7	[10]	0	1	0	0
x_5	M·<-1,-1,0,0>	22	4	4	0	0	1	-1
		22M·<-1,-1,0,0>	4M·<-1,-1,0,0>	4M·<-1,-1,0,0>	0	0	M·<-1,-1,0,0>	M·<1,1,0,0>
			7+8M	13+8M	0	0	0	-2M
x_3	0	83/10	-1/10	0	1	-13/10	0	0
x_2	2·<3,4,2,1>	39/10	7/10	1	0	1/10	0	0
x_5	M·<-1,-1,0,0>	32/5	[6/5]	0	0	-2/5	1	-1
		1/5·<117-32M, 156-32M,78,39>	1/5·<21-6M, 28-6M, 14,7>	2·<3,4,2,1>	0	1/5·<3+2M,4+2M,2,1>	M·<-1,-1,0,0>	M·<1,1,0,0>
			1/10·(24M-21)	0	0	-1/10·(13+8M)	0	-2M
x_3	0	53/6	0	0	1	-4/3	1/12	-1/12
x_2	2·<3,4,2,1>	1/6	0	1	0	1/3	-7/12	7/12
x_1	<2,4,1,3>	16/3	1	0	0	-1/3	5/6	-5/6
		1/3·<35,68, 18,49>	<2,4,1,3>	2·<3,4,2,1>	0	2	1/6·<-18, -1,12,29>	1/6·<1,18,29,12>
			0	0	0	-2	7/4-2M	-7/4

$$\mu_{\tilde{z}_{max}}(x)=\begin{cases}\frac{3x-17}{18}, & \frac{17}{3}<x<\frac{35}{3};\\ 1, & \frac{35}{3}\le x\le\frac{68}{3};\\ \frac{117-3x}{49}, & \frac{68}{3}<x<39;\\ 0, & x\notin(\frac{17}{3},39).\end{cases}$$

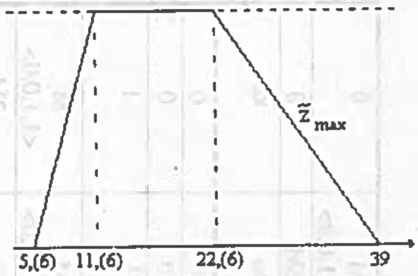

Fig. 3. Graph of the Characteristic Function for PL2 Optimus Solution.

Exampe 5. Non-defined linear program.

In order to obtain product P_1 one uses 2 units from raw material M_1 and one from raw material M_2; product P_2 needs one unit M_1 and 2 units M_2. There are 8 units M_1 and 8 units M_2 in the stock. The unitary benefits are 4 m.u for P_1 and 2 m.u. for P_2. The demand on the market being quite satisfactory, there will be no quantitative restrictions for either product.

Which are the precise quantities that yield maximum profit?

The mathematic model consistent with the question is that in Ratioes (27). Primal Simplex Algorithm applied to the program was presented in table 2.

$$[\max]\, z=4x_1+2x_2$$
$$\begin{cases}x_1+2x_2\le 8\\ 2x_1+x_2\le 8\end{cases}\quad \textbf{(PL1)}\qquad (27)$$
$$x_{1,2}\ge 0$$

After the third iteration x_3 and not x_2 enters the base again. The program curls and goes back to the second iteration: there is an infinity of solution.

Two optimus solution follow from the table:

$$x_1^{'}=4;\ x_2^{'}=0 \text{ şi } \ x_1^{''}=\frac{8}{3};\ x_2^{''}=\frac{8}{3}.$$

The optimus solution set consists of all the convex combinations:

$$x_1^{*}=\frac{12-4\lambda}{3};\ x_2^{*}=\frac{8\lambda}{3},\quad \lambda\in[0,1].$$

and the maximum value of the target function is as follows: $z_{max}=16$.

Table 2.

Primal Simplex Algorithm of the Program in Ratioes (27).

B	C^B	S^B	1	2	0	0	
			x_1	x_2	x_3	x_4	
x_3	0	8	1	2	1	0	8/1=8
x_4	0	8	[2]	1	0	1	8/2=4
		0	0	0	0	0	
			4	2	0	0	
x_3	0	4	0	[3/2]	1	-1/2	8/3
x_1	4	4	1	1/2	0	1/2	8
		16	4	2	0	2	
			0	0	0	-2	
x_2	2	8/3	0	1	[2/3]	-1/3	4
x_1	4	8/3	1	0	-1/3	2/3	-
		16	4	2	0	2	
			0	0	0	-2	

Minding the perturbations caused by the factors of economic instability, PL1 may be associated with a linear program in trapezoidal fuzzy numbers of PL2 type. The example reckons with only the perturbations generated by inflation and subventions over the coefficients of the target function. Thus replacing coefficients 4 and 2 by the trapezoidal fuzzy numbers (3, 5, 3, 1) and (1, 3, 1, 1), respectively, one obtains both the program in trapezoidal fuzzy numbers in Ratioes (28) and the associated one in Ratioes (29):

$$\text{(PL2)}\quad \begin{array}{l}[\max]\,\tilde{z} = (3,5,3,1)x_1 + (1,3,1,1)x_2 \\ \begin{cases} x_1 + 2x_2 \le 8 \\ 2x_1 + x_2 \le 8 \end{cases} \\ x_{1,2} \ge 0 \end{array} \quad (28) \qquad \text{(PL3)}\quad \begin{array}{l}[\max] < \tilde{z} >= 7x_1 + 4x_2 \\ \begin{cases} x_1 + 2x_2 \le 8 \\ 2x_1 + x_2 \le 8 \end{cases} \\ x_{1,2} \ge 0 \end{array} \quad (29)$$

The proper Simplex Algorithm is that in Table 3.

From the table it arises that PL2 and PL3 linear programs have unique optimus solution even through they come from PL1 non-defined program. PL2 optimus solution is the following:

$$x_1^* = \frac{8}{3};\; x_2^* = \frac{8}{3}.\quad \tilde{z}_{max} = \frac{16}{3}(2,4,2,1) = (10,(6);21,(3);10,(6);5,(3)).$$

Table 3.

Primal Simplex Algorithm of Programs (28) and (29).

B	C^B	S^B	<3,5,3,1> x_1	<1,3,1,1> x_2	0 x_3	0 x_4
x_3	0	8	1	2	1	0
x_4	0	8	[2]	1	0	1
		0	0	0	0	0
			7	4	0	0
x_3	0	4	0	[3/2]	1	-1/2
x_1	<3,5,3,1>	4	1	1/2	0	1/2
		4·<3,5,3,1>	<3,5,3,1>	1/2·<3,5,3,1>	0	1/2·<3,5,3,1>
			0	1/2	0	-7/2
x_2	<1,3,1,1>	8/3	0	1	2/3	-1/3
x_1	<3,5,3,1>	8/3	1	0	-1/3·	2/3·
		16/3·<2,4,2,1>	<3,5,3,1>	<1,3,1,1>	1/3·<-3,3,3,5>	1/3·<3,9,7,3>
			0	0	-1/3	-10/3

The characteristic function of the optimus target is given in Figure 4.

$$\mu_{\tilde{z}_{max}}(x) = \begin{cases} \dfrac{3x}{32} & ,0 < x < \dfrac{32}{3}; \\ 1 & ,\dfrac{32}{3} \le x \le \dfrac{64}{3}; \\ \dfrac{80-3x}{16} & ,\dfrac{64}{3} < x < \dfrac{80}{3}; \\ 0 & ,x \notin (0,\dfrac{80}{3}). \end{cases}$$

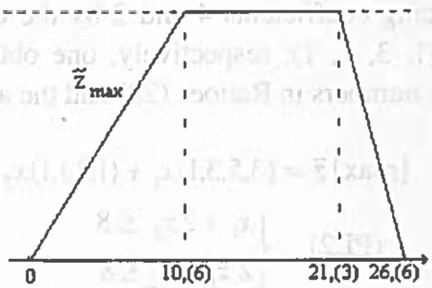

Fig. 4. Graph of the Characteristic Function for PL2 Optimum Solution.

4. Conclusions

Thirty – five after L.A. Zadeh framed the rudiments of fuzzy systems in 1965 it has been found that an important range of scientific fields gain from this particular kind of knowledge. Expert systems, artificial intelligence, robotics, decisional and neural systems are just a few directions of undoubtfully topical interest the approach of which would have been impossible but for the major contribution of fuzzy system theory.

The use of fuzzy systems in modelling the managerial decision process, particularly the theory of multicriterial decisions has gathered momentum.

Today the production planning and the selection models for investments under uncertainty are merely two direction in economics study the grasp of which is intimately related to fuzzy systems. As for the multicriterial analysis the selection of the function to optimise is rendered rather arbitrarily by the traditional viewpoint shared by the operational research.

The models of linear programming with coefficients belonging to the trapezoidal fuzzy number sets ensure a particular flexibility and open new ways of approaching many an economic issue that arises every day under the uncertainty and risk conditions so common to the nowaday Romania.

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