ECONOMICAL SIZE OF FISHING AND PISCICULTURE IN ROMANIA

1. Short history of fishing and pisciculture in Romania

The fish has been, for ages, one of the first economical wealth of this country.

Important historians, among which, Constantin Giurăscu - who was interested in the history and in the problems that the fishing and pisciculture were rising in Romania – wrote in their papers many proofs about fishing activities, the richness in fish of our waters, and the intensive fish trade, mostly at Danube River, at the sea shore and many inner waters and pools (1; p.20).

To prove the old age of these occupations,(among which the historical testimony) comes, also, the Romanian vocabulary, rich in piscicultural terminology, inherited from Latin. Starting with the generic term "fish" (in Romanian "peşte" from "pisces") was formed a family of words: fisher, fisherman, fishing, etc.

At the end of XIXth century the fishing patrimony was of 1.5 million ha; 90% was represented by the Danube's Meadow and its Delta.

The well known Romanian scientist Grigore Antipa said, at the beginning of XXth century that "all the waters that flow through Wallachia along with the Danube River, were producing enormous quantities of fish, surpassing with their productivity all the other waters from Europe"(2; p.18).

This richness in fish of the Danube river and its limitrophe pools has made that even from the ancient times was developed a very important fishing activity and trade with the Greek merchants.

The XIIth century documents are attesting that the fish from the Moldavian ponds was required also by the merchants from Galitia.

If in Romania the fishing, as human occupation, loses its initial point in the darkness of time, as economical activity, it is reported from the feudalism; its scientific basis were placed at the end of XIXth century.

An. Inst. cerc. ec. „G. Zane", t. 10–11, Iaşi, 2001–2002, p. 137–141

The beginning of the next century brings about the construction of a modern Romania and coincides with the start of some actions in order to dyke the Danube's Meadow; the purpose was to increase the agrarian surfaces; until 1933, 59,000 ha were already dyked. This way, besides fishing exploitation of local waters, that was the main activity of that time, it comes the concern for the development of the pisciculture, to breed fish in special arranged precincts, as well as the industrial processing of the fish produces.

This new branch, pisciculture, really enforces itself after 1950–1960 when there are made massive drains especially at the pools from the Danube's Meadow area that represented the first piscicultural potential of this country.

The whole investment effort will be materialized in the construction of numèrous piscicultural nurseries, ponds, pools, accumulation lakes.

The contribution of the scientist work is felt more and more in the process of the development in piscicultural economy and its modernization. The contribution of Grigore Antipa deserves to be reminded: he is known as the first to built the foundation of our modern pisciculture, establishing the coordinates for the structure on its modern principles;he established the fishing and pisciculture activities in Romania, and had a great contribution to the elaboration of the first law of fishing in this country , and through decisive actions, he contributed to revigorate the country's piscicultural potential.

2. Economical size of fishing and pisciculture in Romania until 1990

The first decades of the XXth century has been the age of the modern pisciculture, in the full development and a productive fishing, which provided the fish for the population alimentation.

All the technologies of reproduction, breeding and fishing continued to improve. There were acclimatized new species of fish and, with application of this breeding formulas, the production of 2,000-3,000 kg became usual.

At the same time, this piscicultural field was tributary to the economic policy, centralized and inflexible. This way:

- all investments in piscicol arrangements were promoted after some standards of specific investments, which were low and insufficient to make possible arrangements in conformity with the international standards;
- the endowments supplied by national industry were insufficient and to homologate or to produce systems of machinery proper for pisciculture area was really impossible, because there were difficulties

in producing lines of fabrication for small series, difficulties that determined people's lack of interest, often also shown by industrial units of machines construction;
- fishing equipment import was inaccessible because of non-existence of currency funds;
- the change of technical information was null;
- the breeding of the phytoplanktofagus species extended in excess, on economical grounds, to the prejudice of valuable native species, so that the weight of the first reached, in 1989, a raised level on the whole production;
- the lack of a specialized coordinator on national level made the pisciculture development to be slow comparatively with the existing conditions from our country.

All this time, our passion for pisciculture, the tacit fight against the impostors managed to keep this section unitary. The work done was hard, but ordered.

Starting with 1964, Romania organizes itself a strong ocean-fishing fleet which, in 1989, counts 48 fishing trawlers, 12 fish conveyer vessels, and a ship for fuel transport.

This state policy support for the fishing sector (that was practiced by many developed states) gave some results.

Reducing the concern for the intern production of fish, there are revealed the data published by F.A.O. which places Romania on the first place in the top of the European countries, excepting ex-U.R.S.S., concerning total amount of fish capture in the continental waters between 1983 and 1992, as well as the biggest annual captures for the same period of time.

In addition to these remarkable productions, came the contribution of about 150,000 tone/year captured by the oceanic fishing fleet.

With these productions we are situated between the countries with developed economy: the consume was about 10–12 kg per inhabitant.

3. The size of fishing and the pisciculture in Romania after 1990

The beginning of the 90's represented a turning point for the development of fishing and pisciculture in Romania. The start of transition in the market economy, the changes in the social, political and economical structures, determined some unbalances, that brought an important decrease of fish productions.

The intern production of fish diminished very much, the capture in 1994 was only of 34,000 t fish (comparing with 1989: 77,300 t), because the oceanic fishing was deprived of any help from the state, and had anymore fishing licence, it was not practiced anymore, providing no more raw material for tinned fish industry. All this production once obtained with inner efforts was replaced with the import of oceanic fish, operation which unleashed many speculations and brought prices increase.

This decline was felt on the market, the lack of fish in the organized market, encouraged actions like poaching and the clandestine sale of fish that had like an unjustified result rise of sale prices.

The reasons that lead at this regress are multiple and complex, and connected on the intern level, with problems like:

– excessive exploit of piscicultural pools in the last 20 years;
– aggressive pollution of waters;
– the changes that occurred in the structure of piscicol population by acclimatizing and breeding some new species not wanted by the population;
– the legislative gap existing in transition, stimulating disorders in exploitation;
– the break in the activity of the local fleet.

Deeper roots has the diminution of piscicultural potential in natural waters, Danube and its Delta, resulting from the policy of abusive reduction and non-scientific of reproduction areas for the fish from these waters by building dykes in the Danube's Meadow and rivers meadows or the lack of protection for some valuable species that existed in the inner waters but disappeared in time or remained in small number. This category includes the sturgeon, the blue herring, the plaice and the gray mullet, some species half migratory from the plain rivers from the hill and mountain.

This was the consequence of unfavorable actions practiced in time by a complex of factors, where the man himself had a solid contribution by his actions made without understanding the possible consequences, not knowing the harm he produces.

To stabilize and bring back to normal this new situation, there were initiated by Romanian Fishing and Pisciculture Organization also by the piscicultural trading companies, and the piscicultural research institutions, etc., a number of actions in order to:

– adopt a law for fishing and pisciculture;
– choose a strategy and an unique intention regarding the fishing policy in Romania;

– define and consolidate piscicultural resources management from the waters belonging to public domain, by ecological stabilization and completing the piscicultural stacks, along with the correct sizing of the fishing efforts;to create facilities, to stimulate and encourage the private enterprises or farms that promote modern technologies, acclimatization of performance species or the support for the stack completing action from the natural waters and, the last, trying to produce and to value other aquatic species.
– promote the alignment policy under a technological, technical and managerial aspect, of the piscicultural producers from Romania to the standards practiced in all the countries from European Union;
– support the measures in order to achieve a competition environment, specific for a market economy.

The law regarding the piscicultural back-ground, fishing and aquaculture [4; p.4] carried by Romanian Parliament in 2001, contains some settlements which can contribute at the potential piscicultural protection. The period, the time, the areas that were prohibited, were taken into consideration, also the smallest dimensions for the fish and the other aquatic creatures which can be captured, the smallest dimensions for the meshes of the fishing tools, the conditions that must be carried out for a good practice of angling in the waters belonging to the public domain.

The necessity of the elaboration of a development strategy in fishing and pisciculture in Romania must be added with stages until 2010–2020, with the contribution of all means in order to reorganize the piscicultural economy in all productive units; their stipulations will lead to a prosperous and thriving pisciculture.

References

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UNCERTAINTY MODELLING BY FUZZY NUMBERS

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1. Fuzzy numbers, models of the uncertain information

A set of real numbers is well-defined by its characteristic function:

$$f_A : \mathbf{R} \longrightarrow \{0,1\} \tag{1}$$

Value 0 corresponds to real numbers not belonging to set A and value 1 symbolizes the belonging to the set.

A fuzzy set in **R**, $A \in \mathbf{F(R)}$, will have the characteristic function with values ranging within the whole interval [0,1]:

$$\mu_A : \mathbf{R} \longrightarrow [0,1] \tag{2}$$

An intermediary value between non-belonging 0 and belonging 1 symbolizes the degree (of safety) of the belonging to set A.

A ***fuzzy number***, ($\tilde{a} \in \mathbf{NF(R)} \subset \mathbf{F(R)}$), represents a fuzzy set in **R** with the following properties:

– has as a nucleus a point or a closed interval:

$$N(\tilde{a}) \overset{def.}{=} \{x \in R \mid \mu_{\tilde{a}}(x) = 1\} = [a^m, a^M],\ a^m \le a^M \tag{3}$$

– on each of the left and right portions of the nucleus, the characteristic function is continuous and increasing, respectively decreasing;

– the characteristic function is integrable on **R** (and the graph of the characteristic function has the axis of the abscissas as horizontal asymptote to $\pm\infty$).

The support of a fuzzy number is an open interval (s, d) finite or not to one or both ends:

$$S(\tilde{a}) = \{x \in R \mid \mu_{\tilde{a}}(x) > 0\} \tag{4}$$

The positive numbers $a^s = a^m - s$ şi $a^d = d - a^M$, $a^s, a^d \in \overline{R}_+$ are called the elongation to left and respectively the elongation to right.

Thus, a fuzzy number is a system made of five elements $\tilde{a} = (a^m, a^M; a^s, a^d, \mu_{\tilde{a}})$ with the following properties:

$$
\left.
\begin{array}{l}
, a^m \le a^M \in R \quad , a^s, a^d \in \overline{R}_+ \\
\mu_{\tilde{a}}(x) = \begin{cases} \mu_{\tilde{a}}^s(x) & , a^m - a^s < x < a^m \\ 1 & , a^m \le x \le a^M \\ \mu_{\tilde{a}}^d(x) & , a^M < x < a^M + a^d \\ 0 & , x \notin (a^m - a^s, a^M + a^d) \end{cases} , \mu_{\tilde{a}}^s(x), \mu_{\tilde{a}}^d(x) \in (0,1) \\
\mu_{\tilde{a}}^s(x') < \mu_{\tilde{a}}^s(x'') \quad , \forall x', x'' \ni a^m - a^s < x' < x'' < a^m \\
\mu_{\tilde{a}}^d(x') > \mu_{\tilde{a}}^d(x'') \quad , \forall x', x'' \ni a^M < x' < x'' < a^M + a^d \\
\lim_{\substack{x \to a^m - a^s \\ x > a^m - a^s}} \mu_{\tilde{a}}(x) = 0, \quad \lim_{\substack{x \to a^M + a^d \\ x < a^M + a^d}} \mu_{\tilde{a}}(x) = 0; \\
\exists \int_{-\infty}^{+\infty} \mu_{\tilde{a}}(x)dx = \lim_{t \to \infty} \int_{-t}^{+t} \mu_{\tilde{a}}(x)dx \in R
\end{array}
\right] \quad (5)
$$

The nucleus, as a compact interval, can be made of one point ($a^m = a^M$, degenerated interval), case in which $\tilde{a}$ is called ***sharp fuzzy number.***

Opposingly ($N(\tilde{a}) = [a^m, a^M]$, $a^m < a^M$), $\tilde{a}$ is ***flat fuzzy number***.

The support is a finite or not interval, to one or both ends.

The future economic information is more or less uncertain.

If, for instance, taking into account the calculation of the inflation average of the past two years, one can establish as far as the supply with raw materials for the next trimester is concerned, a monetary fund need of 1000 u.m., it is very unlikely for this information to be the real one. Most likely it is an amount ranging within –5% and +10%. This amount is well represented by a triangular fuzzy number (according to the description of the next paragraph) $\tilde{a} = (1000, 50, 100)$ with the following belonging function:

$$
\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - 950}{50} & , 950 < x \le 1000 \\ \dfrac{1100 - x}{100} & , 1000 < x < 1100 \\ 0 & , x \notin (950, 1100). \end{cases}
$$

2. Classes of fuzzy numbers

Varying with the type of analytical expressions of the belonging functions, the fuzzy numbers can be grouped into different sub-classes.

In practice the most frequently used are the triangular and trapezoidal fuzzy numbers .

A ***fuzzy trapezoidal number*** $\tilde{a} = (a^m, a^M, a^s, a^d) \in \mathbf{Tp(R)}$, (Figure 1.a) has the following characteristic function:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - a^m}{a^s} + 1 & , a^m - a^s < x < a^m \\ 1 & , a^m \le x \le a^M \\ \dfrac{a^M - x}{a^d} + 1 & , a^M < x < a^M + a^d \\ 0 & , x \notin (a^m - a^s, a^M + a^d). \end{cases} \tag{6}$$

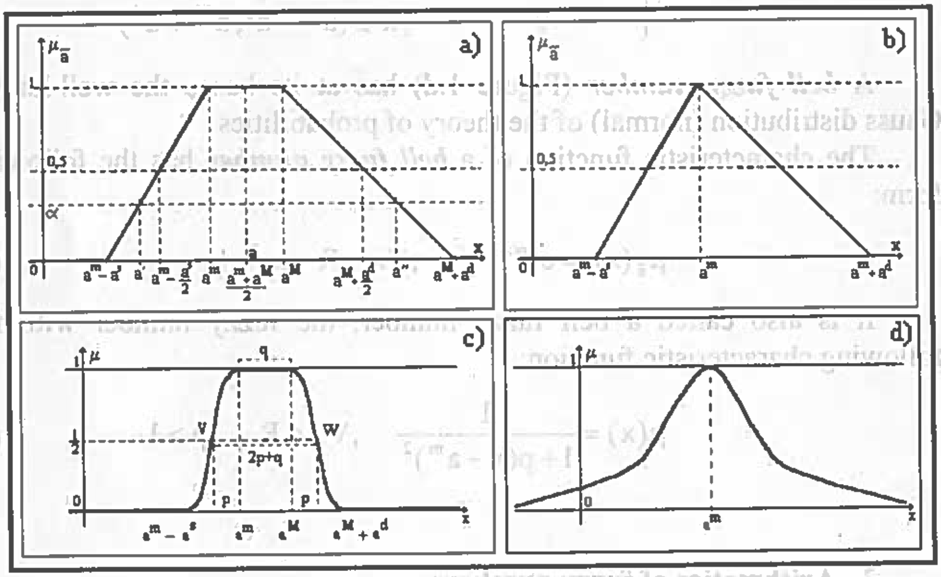

Figure 1. Fuzzy numbers: a) trapezoidal b) triangular c) square d) bell.

A sharp fuzzy trapezoidal number ($a^m = a^M$) is called ***fuzzy triangular number*** being symbolized by a third form $\tilde{a} = (a^m, a^s, a^d)$ (Figure 1.b).

The degenerated triangular fuzzy number $(a^m, 0, 0)$ corresponds to the real number a^m.

A ***square fuzzy number*** (Figure 1.c) has the restriction of the characteristic function on the two portions (intervals) of different monotonies, made of parabolas:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{(x - a^m + a^s)^2}{2(a^s - p)^2} & , a^m - a^s < x < a^m - p \\ 1 - \dfrac{(x - a^m)^2}{2p^2} & , a^m - p \le x < a^m \\ 1 & , a^m \le x \le a^M = a^m + q \\ 1 - \dfrac{(x - a^M)^2}{2p^2} & , a^M < x < a^M + p \\ \dfrac{(x - a^M - a^d)^2}{2(a^d - p)^2} & , a^M + p < x < a^M + a^d \\ 0 & , x \notin (a^m - a^s, a^M + a^d) \end{cases} \quad (7)$$

A ***bell fuzzy number*** (Figure 1.d) has at its bases the well-known Gauss distribution (normal) of the theory of probabilities.

The characteristic function of a ***bell fuzzy number*** has the following form:

$$\mu_{\tilde{a}}(x) = e^{-p(x-a^m)^2} \quad , \forall x \in R \quad , p > 1 \quad (8)$$

It is also called a bell fuzzy number, the fuzzy number with the following characteristic function:

$$\mu(x) = \frac{1}{1 + p(x - a^m)^2} \quad , \forall x \in R \quad , p > 1 \quad (9)$$

3. Arithmetics of fuzzy numbers

Taking into account that fuzzy numbers represent mathematic models of the uncertain numeric information, one can easely raise the issue of operation definition with these numbers.

Taking two fuzzy numbers: $\tilde{a}=(a^m,a^M,a^s,a^d,\mu_{\tilde{a}})$ and $\tilde{b}=(b^m,b^M,b^s,b^d,\mu_{\tilde{b}})$.

The addition of the fuzzy numbers $\tilde{a}+\tilde{b}$ is defined as follows:

$$\left.\begin{array}{l}\tilde{a}+\tilde{b}=(a^m+b^m,a^M+b^M,a^s+b^s,a^d+b^d,\mu_{\tilde{a}+\tilde{b}})\\ \mu_{\tilde{a}+\tilde{b}}(x)=\begin{cases}\left[\mu s_{\tilde{a}}^{-1}+\mu s_{\tilde{b}}^{-1}\right]^{-1}(x) & ,x\in(s_1,d_1)\\ \left[\mu d_{\tilde{a}}^{-1}+\mu d_{\tilde{b}}^{-1}\right]^{-1}(x) & ,x\in(s_2,d_2)\\ 1 & ,x\in[d_1,s_2]\\ 0 & ,x\notin(s_1,d_1)\end{cases}\\ \text{where}: d_1\overset{not.}{=}a^m+b^m,\quad s_1\overset{not.}{=}d_1-a^s-b^s,\\ \qquad s_2\overset{not.}{=}a^M+b^M,\quad d_2\overset{not.}{=}s_2+a^d+b^d\end{array}\right] \quad (10)$$

The multiplication of a fuzzy number $\tilde{a}=(a^m,a^M,a^s,a^d,\mu_{\tilde{a}})$ ***with a scalar*** $t\in R$ makes a new fuzzy number $t\tilde{a}$ defined as follows:

$$\left.\begin{array}{l}t\tilde{a}=\begin{cases}(ta^m,ta^M,ta^s,ta^d,\mu_{t\tilde{a}}) & ,t>0\\ (ta^M,ta^m,-ta^d,-ta^s,\mu_{t\tilde{a}}) & ,t<0\\ (0,0,0,0,\mu_{\tilde{0}}) & ,t=0\end{cases}\\ \text{where}:\quad \mu_{t\tilde{a}}(x)=\mu_{\tilde{a}}\left(\frac{x}{t}\right),\forall x\in R\ ,(\forall)t\in R\setminus\{0\}\end{array}\right] \quad (11)$$

The opposite of the fuzzy number $\tilde{a}$ marked $-\tilde{a}$ is obtained as follows:

$$-\tilde{a}\overset{def}{=}(-1)\cdot\tilde{a}=(-a^M,-a^m,a^d,a^s,\mu_{-\tilde{a}}),\ \ \mu_{-\tilde{a}}(x)=\mu_{\tilde{a}}(-x)\ \ ,\forall x\in R \ (12)$$

The addition of the fuzzy numbers is associative and commutative and the triangular fuzzy number (real) $\tilde{0}$=(0,0,0) is a neutral element for addition. Generaly, the addition between a fuzzy number and its opposite does not have as result $\tilde{0}$:

$$\left.\begin{array}{c}(\tilde{a}+\tilde{b})+\tilde{c}=\tilde{a}+(\tilde{b}+\tilde{c})\\ \tilde{a}+\tilde{b}=\tilde{b}+\tilde{a}\\ \tilde{a}+\tilde{0}=\tilde{0}+\tilde{a}=\tilde{a}\end{array}\right]\ ,\forall\tilde{a},\tilde{b},\tilde{c}\in NF(R) \quad (13)$$
$$\tilde{a}+(-\tilde{a})=\tilde{0}\Leftrightarrow a^s=a^d=0\Leftrightarrow\tilde{a}=(a^m,0,0)\in R$$

The set of fuzzy numbers with the addition operation (**NF(R)**, +), makes an algebric structure of ***monoid additive commutative.***

The real number associated to a fuzzy number $\tilde{a} = (a^m, a^M, a^s, a^d, \mu_{\tilde{a}})$, marked $< \tilde{a} > \in R$, is calculated with the following relations:

$$\left.\begin{array}{l} < \tilde{a} > \overset{def.}{=} \dfrac{a^m + a^M}{2} - As_{\tilde{a}} + Ad_{\tilde{a}} \\ \text{where: } As_{\tilde{a}} = \displaystyle\int_{a^m - a^s}^{a^m} \mu_{\tilde{a}}(x)dx; \quad Ad_{\tilde{a}} = \int_{a^M}^{a^M + a^d} \mu_{\tilde{a}}(x)dx \end{array}\right] \qquad (14)$$

The two integrals represent the area of the surfaces between graph $\mu_{\tilde{a}}$ and the axes of the abscissas, areas situated to the left and respectively to the right of the nucleus.

With the help of the three notions (addition, multiplication with scalar and associated real number) one can defined the other operations as well as an order relation:

$$\left.\begin{array}{ll} \textbf{\textit{a) substraction:}} & \tilde{a} - \tilde{b} \overset{def.}{=} \tilde{a} + (-\tilde{b}) = \tilde{a} + (-1)\cdot\tilde{b} \\ \textbf{\textit{b) multiplication:}} & \tilde{a}\tilde{b} \overset{def.}{=} \dfrac{\tilde{a} < \tilde{b} > + < \tilde{a} > \tilde{b}}{2} \\ \textbf{\textit{c) reverse:}} & \tilde{b}^{-1} \overset{def.}{=} \dfrac{\tilde{b}}{\langle \tilde{b} \rangle^2} \\ \textbf{\textit{d) division:}} & \dfrac{\tilde{a}}{\tilde{b}} \overset{def.}{=} \tilde{a}\cdot\tilde{b}^{-1} = \dfrac{\tilde{a}\tilde{b}}{< \tilde{b} >^2} = \dfrac{\tilde{a} < \tilde{b} > + < \tilde{a} > \tilde{b}}{2 < \tilde{b} >^2} \\ \textbf{\textit{e) degree increase:}} & \tilde{a}^n \overset{def.}{=} \tilde{a} < \tilde{a} >^{n-1}, \forall n \in Q \\ \textbf{\textit{f) order relation:}} & \tilde{a} > \tilde{b} \quad \Leftrightarrow \quad < \tilde{a} > \; > \; < \tilde{b} > \\ \textbf{\textit{g) resembling relation:}} & \tilde{a} \cong \tilde{b} \overset{def.}{\longleftrightarrow} < \tilde{a} > = < \tilde{b} > \end{array}\right] \qquad (15)$$

The definitions were conceived so that to preserve some important properties of the homologous operations with real numbers. Out of these one can remind:

$$\left.\begin{array}{l} <t\tilde{a}> = t<\tilde{a}>; \quad t(s\tilde{a}) = (ts)\tilde{a}; \quad t(\tilde{a}+\tilde{b}) = t\tilde{a}+t\tilde{b}; \\ <\tilde{a}\pm\tilde{b}> = <\tilde{a}> \pm <\tilde{b}>; \quad \left\langle \sum_{k=1}^{n} t_k\tilde{a}_k \right\rangle = \sum_{k=1}^{n}(t_k<\tilde{a}_k>); \\ <\tilde{a}\cdot\tilde{b}> = <\tilde{a}>\cdot<\tilde{b}>; \quad \left\langle \frac{\tilde{a}}{\tilde{b}} \right\rangle = \frac{<\tilde{a}>}{<\tilde{b}>}; \quad \left\langle \tilde{b}^{-1} \right\rangle = \left\langle \tilde{b} \right\rangle^{-1}; \\ \left\langle \sqrt[m]{\tilde{a}^n} \right\rangle = \sqrt[m]{<\tilde{a}>^n}; \\ \tilde{a}^n\tilde{a}^m = \tilde{a}^{n+m}; \; (\tilde{a}^n)^m = \tilde{a}^{nm}; \; \frac{\tilde{a}^n}{\tilde{a}^m} = \tilde{a}^{n-m}; \; (\tilde{a}\tilde{b})^n = \tilde{a}^n\tilde{b}^n; \; \left(\frac{\tilde{a}}{\tilde{b}}\right)^n = \frac{\tilde{a}^n}{\tilde{b}^n}. \end{array}\right] \quad (16)$$

The order relation defined in (15.f) is not a relation of total order.

Still, some very important properties of the order relation of the real numbers are true for the order relation with fuzzy numbers, too.

For instance:

$$\tilde{a} > \tilde{b} \quad \Rightarrow \quad \left[\begin{array}{ll} \tilde{a}\pm\tilde{c} > \tilde{b}\pm\tilde{c} \quad ,\forall\tilde{c} & \\ \tilde{a}\tilde{c} > \tilde{b}\tilde{c} \;,\forall\tilde{c}>0 & \tilde{a}\tilde{c} < \tilde{b}\tilde{c} \;,\forall\tilde{c}<0 \\ \frac{\tilde{a}}{\tilde{c}} > \frac{\tilde{b}}{\tilde{c}} \;,\forall\tilde{c}>0 & \frac{\tilde{a}}{\tilde{c}} < \frac{\tilde{b}}{\tilde{c}} \;,\forall\tilde{c}<0 \end{array}\right. \quad (17)$$

The resemblance relation defined in (15.g) is reflexive, symmetrical and transitive being in fact a relation of equivalence:

$$\tilde{a} \cong \tilde{a}; \quad \tilde{a} \cong \tilde{b} \Rightarrow \tilde{b} \cong \tilde{a}; \quad \left.\begin{array}{l} \tilde{a} \cong \tilde{b} \\ \tilde{b} \cong \tilde{c} \end{array}\right] \Rightarrow \tilde{a} \cong \tilde{c} \quad (18)$$

The space of this relation of equivalence $NF(R)/\cong$ is isomorphous with the set of real numbers:

$$\left.\begin{array}{l} NF(R)/\cong \overset{def.}{=} \{\hat{r}\}_{r\in R}; \quad \hat{r} \overset{def.}{=} \{\tilde{a} \in NF(R) \mid <\tilde{a}> = r\} \\ NF(R)/\cong \overset{f.bij.}{\longleftrightarrow} R \\ \bigcup_{r\in R} \hat{r} = NF(R); \quad \hat{r}_1 \cap \hat{r}_2 = \Phi \quad ,\forall r_1 \neq r_2 \end{array}\right] \quad (19)$$

Any fuzzy number of an equivalence class can be considered to be representative of the respective class, but, ussualy it is preferred as representative the pure real number r (unique) contained too, in the respective class.

Bijection f defining the isomorphism between the space as well as R preserves all the operations previously defined:

$$\left.\begin{array}{c} f(\hat{r}_1 \circ \hat{r}_2) = r_1 \circ r_2 \\ \hat{r}_1 > \hat{r}_2 \xleftrightarrow{\text{def.}} r_1 > r_2 \end{array}\right] \tag{20}$$

Therefore, the space as long as it has a complex algebric structure of total settled commutative body (and of vectorial space).

Taking into account that in applications the trapezoidal fuzzy numbers are preferred (and the sub-class of triangular fuzzy numbers) one will particularize the definitions of the addition and of the associated real number for these classes of fuzzy numbers:

$$\left.\begin{array}{c} \tilde{a} + \tilde{b} = (a^m + b^m, a^M + b^M, a^s + b^s, a^d + b^d) \\ <\tilde{a}> = \dfrac{a^m + a^M - a^s + a^d}{2} \end{array}\right], \forall \tilde{a}, \tilde{b} \in \mathbf{Tp(R)} \tag{21}$$

$$\left.\begin{array}{c} \tilde{a} + \tilde{b} = (a^m + b^m, a^s + b^s, a^d + b^d) \\ <\tilde{a}> = a^m + \dfrac{a^d - a^s}{2} \end{array}\right], \forall \tilde{a}, \tilde{b} \in \mathbf{Tr(R)} \tag{22}$$

Another great benefit of the way which the operations were introduced, is the fact that some ***internal operations*** on **Tp(R)** and respectively on **Tr(R)**.

4. Classic and fuzzy models

Formally, a ***classic model*** $M = (p, Z_p, A_p)$ is a third form made of the vector of the entrance parameters $p \in R^k$, optimized function (max/min) $Z_p : R^m \longrightarrow R$ and the system of admission conditions (made of a system of m equality and inequality with and unknown, usually non-linear): $A_p(x) \overset{\leq}{\underset{\geq}{=}} 0, x \in R^n$.

The set of admissible solutions and the set of optimal solutions are:

$$S^A = \left\{ x \in R^n \mid A_p(x) \begin{matrix}\geq\\=\\\leq\end{matrix} 0 \right\} \text{ and } S^O = \left\{ x^\bullet \in R^n \mid Z_p(x^\bullet) \begin{matrix}\geq\\\leq\end{matrix} Z_p(x), \forall x^\bullet \in S^A \right\}.$$

Two models M_1 and M_2 are ***the equivalent models*** if they have the same sets of admissible and optimal solutions: $M_1 \leftrightarrow M_2 \begin{cases} S_1^A = S_2^A \\ S_1^O = S_2^O \end{cases}$.

A ***fuzzy model*** $\widetilde{M} = (\widetilde{p}, Z_{\widetilde{p}}, A_{\widetilde{p}})$ is a model for which all the entrance parameters are fuzzy numbers, and to this one can associate a ***classic model*** $\langle \widetilde{M} \rangle = (\langle \widetilde{p} \rangle, Z_{\langle \widetilde{p} \rangle}, A_{\langle \widetilde{p} \rangle})$ which is obtained by replacing in the fuzzy model all fuzzy numbers with associated real numbers.

A very important result is the one given by the following theorem:

Theorem (of fuzzy model charaterization).

Any fuzzy model is equivalent with its associate model.

The characterization of the fuzzy models allows as to precise the stages that have to be followed for solving a fuzzy model:

– the classic model is associated to the fuzzy model by replacing all fuzzy numbers with their associate real numbers;
– the classic model associated is solved by methods specific to any type of model. The literature in the field is very generous in this respect. The optimal solutions of this model (classic) are thus obtained;
– the optimal solution(s) determined in the previous stage is substituted in the function to be optimized (fuzzy) thus obtaining, besides the optimal solution(s), the optimal value, too: $\widetilde{v}^* = Z_{\widetilde{p}}(x^*)$.

An example for solving a fuzzy model is present in the article "*Determination of optimal ways in fuzzy graphs*" within the hereby publication.

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