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SELECTING THE VARIANTS FOR INVESTMENTS UNDER UNCERTAINTY CONDITIONS

1. Introduction

The problems of selecting the variants for investments under uncertainty conditions is very complex, due to the multitude of quantitative and especially qualitative factors which determines it. Certain quantitative factors are easily to be measured, but most factors are difficult to be measured, and certain qualitative factors can not even be evaluated.

This paper approaches the complexity of these problems with the aid of the subtle variables which are both mathematically and economically defined.

We introduced two antagonistic concepts, namely, the impulse for investments and the inhibition for investments which allow the selection of the variants in the conditions of the multitude of qualitative and quantitative factors. We have in view the fact that the methodology of operativelly adopting the decisions with the aid of the subtle variables uses antagonistic concepts.

2. Subtle variables

We call subtle variables an indicator expressed by natural language which can be characterized with the aid of a large number of quantitative, qualitative, hidden and/or apparent factors which influence it in a favourable way, up to an optimum threshold (respectively, in an unfavourable way up to the limit of a non-optimum threshold). The number of factors can tend to infinite, and from a mathematical point of view, if it is advantageous for the model, we can consider even a continuum of factors. This paper deals with the finite case. Initially, because of the lack of complete information, one considers that all apparent factors are true. Subsequently, while receiving more information and knowledge, the apparent levels are corrected.

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We denoted by S the subtle variable and by $f_1, f_2, \ldots f_n$ the set of n influence factors taken into consideration. The $(f_1,\ldots,f_m)$ quantitative factors are divided into two categories: indefinite factors, which are fuzzy variables and definite factors , which are considered deterministic variables. The $(f_{m+1},\ldots,f_n)$ qualitative factors are linguistic type variables (*fuzzy*).
In these conditions, the subtle variable is represented as follows:

$$S=(f_1,f_2,\ldots,f_m,f_{m+1},\ldots,f_n) \quad (1)$$

The linguistic factors are characterized both with the aid of comparison degrees and with the aid of marks, on several scales: 1..10, 1..5, etc.
We consider a set of E elements (subjects, variants etc.) For each element of e rank belonging to E set, we can outline the levels of $f_{1e},f_{2e},\ldots f_{me}$ quantitative factors, as well as that of $f_{m+1e},\ldots,f_{ne}$ qualitative factors.
The levels of the subtle variables attached to *e* element are:

$$S_e=(f_{1e},f_{2e},\ldots,f_{me},f_{m+1e},\ldots,f_{ne}) \quad (2)$$

We can define a set of elements attached to non-optimum levels, under the form:

$$S_{pes}=(f_{1pes},f_{2pes},\ldots,f_{m\ pes},f_{m+1\ pes},\ldots,f_{n\ pes}) \quad (3)$$

At the same time, a set attached to optimum levels is defined, under the form:

$$S_{opt}=(f_{1\ opt},f_{2\ opt},\ldots,f_{m\ opt},f_{m+1\ opt},\ldots,f_{n\ opt}) \quad (4)$$

The two sets defined above (non-optimum and optimum) are conventional.
One defines a distance between the S_e subtle set and the S_{opt} conventional set, denoted by $\delta\ (S_e,S_{opt})$, as well as the distance between the subtle set and the S_{pes}, denoted by $\delta\ (S_e,S_{pes})$.
If we want to optimize S_e, it is necessary that $\delta(S_e,S_{opt})$ to be minimum.
Further on, it is defined a δ dominant relationship between two subtle sets attached to *e* and *e'* elements.
Then:

$$S_e\ D\ S_{e'} \quad (5)$$

If $f_{ie} \geq f_{ie'}$, for *i* factor, if we want its maximization, respectively $f_{ie} \leq f_{ie'}$, if we want its minimization (at least one relationship being broken). The indifference relationship *J* is obtained if :

$$S_e\ D\ S_{e'} \text{ is false and } S_{e'}\ D\ S_e \text{ is false} \quad (6)$$

The relationships *D* and *J* allow a partial ordering of the subtle set.
If we define the $\delta(S_e,...S_{opt})$ distance which is a scalar, we can obtain a total ranking of the subtle set. To this end, we introduced a *P* preference relationship between the *e* and *e'* elements, according to the condition:

$$S_e \; P \; S_{e'}, \text{ if } \delta(S_e, S_{e\,opt}) < \delta(S_{e'}, S_{e\,opt}) \tag{7}$$

The $\delta(S_e,S_{opt})$ distance is defined as a relative Hamming distance, relative Euclidian distance etc., where the significance factors can be weighted with respect to the significance (the structure of factors significance being another subtle variable). There are decision-makers which consider, for simplification, that all factors are equally important.

The distance is easily calculated if one transforms the subtle variable into a scalar, for instance, into an utility.

Given a *j* element (variant) and an *i* factor, the u_{ij} utility is given by:

$$u_{ij} = \frac{f_j - f_{ipes}}{f_{iopt} - f_{ij}} \tag{8}$$

In case of an operator of additive composition, the U_j overall utility of *j* variant is:

$$U_{ja} = \sum_{i=1}^{n} k_i u_i \tag{9}$$

k_i = normalized significance coefficients (selected by group decision);

At the same time, we can use a multiplicative composition law:

$$U_{jm} = \prod_{i=1}^{n} u_{ij}^{ki} \tag{10}$$

where U_{ja} = overall utility calculated *via* additive method;

U_{jm} = overall utility calculated *via* multiplicative method.

If there is a stochastic dependence between the f_i and f_n factors, whose correlation coefficient is p_{ih}, then, the k_i significance coefficient can be corrected, as follows:

$$k_i = (1-p_{ih})k_i \tag{11}$$

It is noticed that the new k_i coefficients will be no more normalized, consequently, the operation of renormalizing these coefficients is necessary. As a result of renormalization, one obtains new k_i^* coefficients. In (9) or in (10), k_i is replaced by k_i^*.

The $\delta(S_e,S_{opt})$ distance is calculated with the aid of the relationship:

$$\delta\,(S_e,S_{opt}) = 1\text{-}U_{es} \qquad (12)$$

Generally, it is possible that for the factors which define a subtle variable, to exist more correlations between two factors.

Particularly, if we exclude the p_{ii} autocorrelations by time lag δ_i, we shall obtain a matrix $(n^2\text{-}n)$ with arcs (elements).

If there are autocorrelations with a certain time lag Δ, a graph with n square arcs is obtained, respectively, a matrix with n square elements.

The two order relevance (for two factors) R_{i2} (a factor h with respect to a factor i) is:

$$R_{i2} = \sum_{i=1}^{n} \rho_{ij} + \sum_{j=1}^{n} \rho_{ij} - \rho_{ii} \qquad (13)$$

We can order the f_i factors according to the R_{i2} elements and one can verify the Paretto law (respectively, the division of factors into A,B and C classes).

If R_{i2} is low with respect to other relevance factors, then, the f_i factor can be neglected.

For a more correct comparison of the absolute relevances, one recommends to do their normalization, case where R^*_{i2} normalized relevance coefficients are obtained.

In this case, we can introduce a threshold belonging to (01) interval, where the factors to be taken into consideration must be included.

For more refined analyses, a three order relevance is defined, whose coefficients will be denoted by R_{i3}. The graph projected for this purpose will be obtained *via* Latin multiplication of the above mentioned matrices. Analogously, the aggregated factors whose relevance exeeds a certain threshold will be obtained.

We consider, that, in general, the two order relevance ensures the accuracy of the present requirements.

3. The impulse for investments

The impulse for investments for certain V_j investment variants is made up of the totality of factors which contribute to taking the decisions for assigning financial resources, with a view to achieve that variant. Among them, we can take into consideration those ones which intuitivelly are considered as relevant, namely:

- demand of products achieved in V_j variant;
- production capacity achieved by V_j variant with respect to the demand (the degree to which it satisfies the demand, as regards the quantitative and qualitative aspect);
- investments volume and investments achievement scheduling;
- profit and currency input;
- cutting down specific consumptions (especially, scarce resources);
- pollution degree;
- ethical conditions (as a rule, very hidden), directly known or intuitivelly deduced;
- ergonomic and aesthetic conditions;
- social conditions (supply of working places, working conditions);
- turning to good account the earned experience;
- ensurence of enterprise future development
- conditions of total management of quality.

We notice that some of the factors mentioned above are, in their turn, subtle variables, but they can be divided into elementary factors, fuzzy or deterministic variables. Numerous factors can be differently regarded by the company and state administration, as it is the case of the pollution degree, ethic conditions, ergonomic conditions, etc. In such cases, the companies create an appearance in the spirit of the legislation, consequently, they falsify the correct data. By composing these factors, using relationships (9) and (10), one finds out which are the variants whose utility exceeds a threshold accepted by the decision-maker which gives the impulse necessary to investing decisions.

4. Inhibition for investments

The inhibition for investments of V_j variant represents the totality of factors which oppose to the investing decision, and can be mathematically expressed by the multitude of cases where the thresholds of the indicators generating the impulse for investing are not reached.

Consequently, in order to determine the inhibition for investments, one calculates the most unfavourable possible levels of the factors determining the impulse for investing and one outlines the different possible causes of certain situations responsible for failure (hostile business environment, corruption, economic blocking, inflation, etc differentiated according to the selected V_h variant). For instance:

- strong competition in the market of products and services, but also in the market of raw materials and materials;
- technical progress and transfer of technology;
- delays caused by organizational culture;
- rapid inflation;
- country risk.

Obviously, if such a cause has a constant level for all V_h variants, the less relevant ones will be eliminated.

5. Conclusions

The two antagonistic concepts mentioned above (impulse and inhibition for investments) and the use for each variant selected according to success criteria of the mathematical model concentrated in relationships (9), (10) and (12), represent the fundamentals of practical application of a new decision-making technique in investments.

To this end, one calculates the difference between the determined antagonistic indicators and one selects the variant with the highest difference (high impulse, low inhibition).

In addition, the subtle sets will allow that for an enough long period of time, a base of knowledge to be created which to be able to make a difference between the real economy and the hidden one.

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