SIMULATION OF THE SIMPLE AND IRREDUCIBLE PRODUCTION SYSTEMS

1. Introduction

The latest researches in the cybernetics of the industrial systems point out certain relations between the quantities of Fig. 1, where:

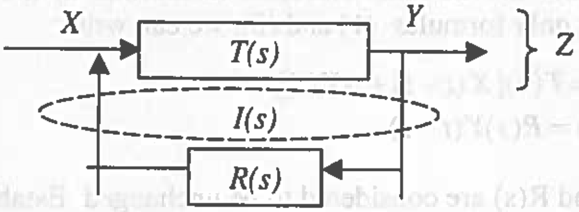

Fig. 1

X = inputs vector (materials, energy, information, manpower, financial resources etc.)

T(s) = transformation structure

I(s) = information system

R(s) = regulation – control system

The relation sequences that can be written according to diagram 1 are as follows:

$$Y = T(s)\,[X + \Delta X] \tag{1}$$

$$\Delta X = Y \cdot R(s) \tag{2}$$

$$Y = T(s) \cdot [X + Y \cdot R(s)] \tag{3}$$

thus resulting:

$$Y = \frac{T(s)}{I - T(s) \cdot R(s)} X \tag{4}$$

or $$Y = G(s) \cdot X \quad (5)$$

where: $$G(s) = \frac{T(s)}{I - T(s) \cdot R(s)} \quad (6)$$

I = identity matrix.

2. Simulation of the irreducible systems

Considering the simulation systems as being in accordance with the description of Edmond Nicolau, namely as iterative process, and considering that the calculus elements mentioned in the relation (4) are scalars, using only formulas (1) and (2), we can write:

$$Y(t) = T(s)[X(t-I) + \Delta X(t)] \quad (7)$$
$$\Delta X(t) = R(s)Y(t-I) \quad (8)$$

where T(s) and R(s) are considered to be unchanged. Establishing the values of T(s) at 0.75, of R(s) at 0.65, of X(1) at 0.9 and of ΔX(0) at 0, in conformity with formulas (7) and (8), accordingly adapted, so that:

$$Y(t) = T(s)[X(t) + \Delta X(t-I)] \quad (9)$$

$$\begin{cases} \Delta X(t) = R(s)Y(t-1) \\ X(t+1) = X(t) + \Delta X(t)\Delta X(t) = R(s)Y(t-1) \\ \Delta X(0) = 0 \end{cases} \quad (10)$$

we will obtain, through simulation, the following conduct of the described system:

Table 1.

Simulation of the system described by (10)

t	*X(t)*	*Y(t)*	*ΔX(t)*	*X(t+1)*
1	0,900	0,675	0,439	1,339
2	1,339	1,333	0,866	2,205
3	2,205	2,303	1,497	3,702
4	3,702	3,899	2,534	6,236
5	6,236	6,577	4,275	10,511

A brief analysis of the data in the table above will immediately show that the simulated process has an exponential development (according to diagram 2) which, should there be no natural limits, could develop itself infinitely.

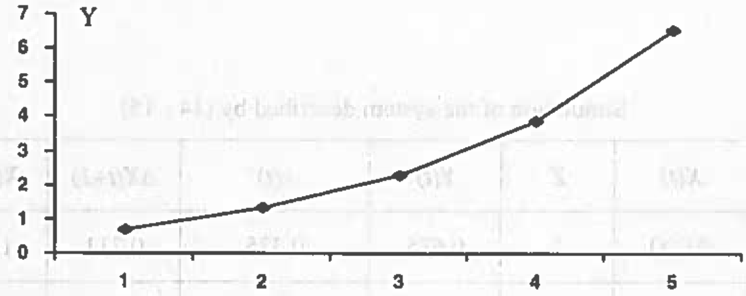

Fig. 2

These natural limits must result from the structure of the system itself, so that, should we try to render the cybernetic system as in fig. 3, where there appears the difference:

$$\varepsilon = Z - Y \tag{11}$$

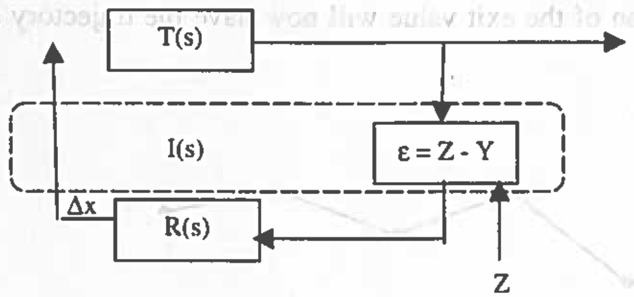

Fig. 3

between the achievements Y and the objectives Z (difference seized and determined by the I(s) information system); the functionals that describe, in this case, the conduct of the newly considered system, are:

$$Y = T(s)[X + \Delta X] \tag{12}$$

$$\Delta X = R(s) \cdot \varepsilon \tag{13}$$

and which, described and ordered with regard to time, are as follows:

$$Y(t) = T(s)[X(t-1) + \Delta X(t)] \quad (14)$$

$$\Delta X(t) = R(s)[Z - Y(t-1)] \quad (15)$$

For Z = 1 the simulation of the systems, according to the formulas above, becomes:

Table 2.

Simulation of the system described by (14 - 15)

t	*X(t)*	*Z*	*Y(t)*	*ε(t)*	*ΔX(t+1)*	*X(t+1)*
1	0,900	1	0,675	0,325	0,211	1,111
2	1,111	1	0,991	0,009	0,006	1,117
3	1,117	1	0,842	0,158	0,103	1,220
4	1,220	1	0,992	0,008	0,005	1,225
5	1,225	1	0,923	0,077	0,05	1,275

The evolution of the exit value will now have the trajectory described in fig. 4.

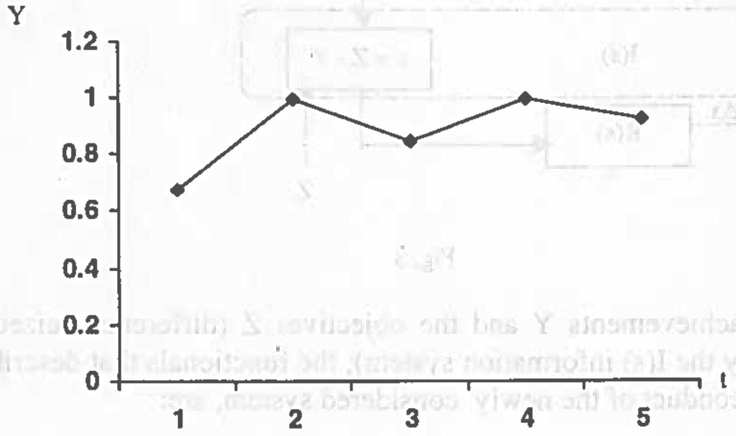

Fig. 4

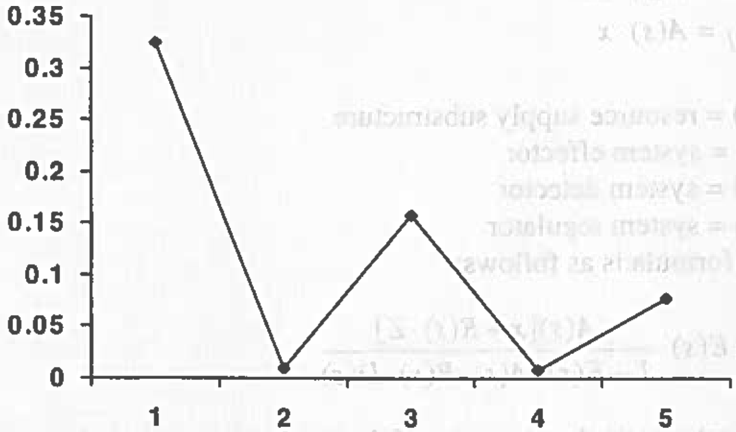

Fig.5

Both the calculations and the diagram suggest an **ergodic process**, with oscillations reduced (absorbed) due to the regulation process.

Taking into account the above basic methods, we can develop and build other models, specific for the production systems.

As a natural consequence of the analysis, the next step is the study of the 4 elements – system (according to fig. 6):

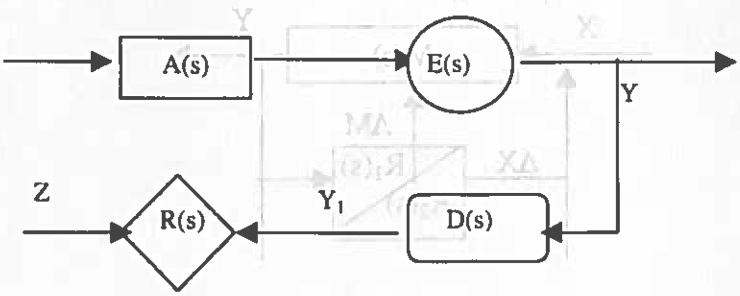

Fig. 6

The functionals that describe the evolution of the system above are rendered through the following formulas:

$$Y = E(s)[x_I + \Delta x] \tag{16}$$

$$Y_I = D(s) \cdot Y \tag{17}$$

$$\Delta Z = R(s)[Z - Y_I] = R(s)[Z - D(s) \cdot Y] \tag{18}$$

$$\Delta x = A(s) \cdot \Delta Z \quad (19)$$

$$x_1 = A(s) \cdot x \quad (20)$$

where:

A(s) = resource supply substructure
E(s) = system effector
D(s) = system detector
R(s) = system regulator

The final formula is as follows:

$$Y = E(s) \cdot \frac{A(s)[x + R(s) \cdot Z]}{1 - E(s) \cdot A(s) \cdot R(s) \cdot D(s)} \quad (21)$$

The schematic development of the production systems' representations can continue, but, from the practical point of view, it is less recommended, given the supplementary difficulties that can arise in the case of such developments; given these reasons, the simulation of the conduct of the production systems is realized on simpler models, that can be more useful and suggestive.

Thus, should we compare the **work place to an irreducible production system**, the latter could be graphically represented as below:

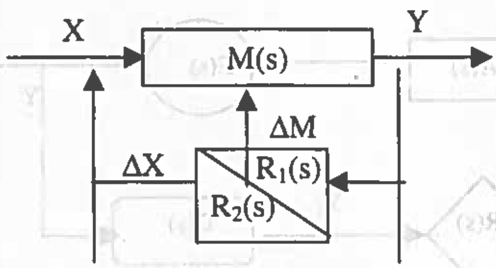

Fig. 7

where:
X = number of inputs
M(s) = state – transformation matrix of the equipment (production capacity)
Y = number of exits
$R_1(s)$ = human factor associated to the technical – technological adjustment
$R_2(s)$ = human factor associated to the supply processes' regulation
ΔX = modification affecting the number of inputs

ΔM = modifications of the machine's state.

In order to simplify the calculations, we will consider all the specified matrixes to be scalars, and therefore we will use the following notation system, in univocal correspondence with the above-mentioned notations: x, m, y, r_1, r_2, d_x and d_m.

Should we consider r as standing for the total regulation capacity of the human factor, then:

$$r_2 = r - r_1 \quad (22)$$

$$y = (m+dm)(x+dx) \quad (23)$$

$$dm = r_1 \cdot y;\ dx = yr_2 = (r - r_1)y \quad (24)$$

We also introduce two supplementary notations:

m_0 = transformation capacity of the machine (equipment)

u_0 = entrances that are not depending on human factors

The formula is now:

$$m = m_0 + k \cdot dm \quad (25)$$

where k = dimensionless coefficient, designating a measure of the capacity of the equipment of being influenced by the human factor; the formula (23) becomes then:

$$y = m_0 \cdot x + [m_0(r - r_1) + k \cdot r_1 \cdot x]y + k \cdot r_1(r - r_1)y^2 \quad (26)$$

and if we note:

$$a = k \cdot r_1(r - r_1)$$
$$b = m_0(r - r_1) + k \cdot r_1 \cdot x - 1 \quad (27)$$
$$c = m_0 \cdot x$$

we will obtain:

$$f(y) = ay^2 + by + c = 0 \quad (28)$$

that will have a maximum for:

$$\frac{df(y)}{dy} = 0 \text{ resulting } y_{opt} = -\frac{b}{2a} \quad (29)$$

or:

$$y_{opt} = -\frac{m(r-r_1)+kr_1x-1}{2kr\cdot(r-r1)} = \frac{mr_1-mr-kr_1x+1}{2kr_1(r-r_1)} \quad (30)$$

It is obvious that, in order to obtain a value as high as possible for y, it is imperative that $a \to 0$ or $b<0$, or, with another formula:

$$\begin{cases} kr_1(r-r_1) \to 0 \\ m_0(r-r_1) + kr_1x - 1 < 0 \end{cases} \quad (31)$$

We will have:

- $a \to 0$ if $r_1 \to 0$
- $b < 0$ if $r_1 = 0$

which means that the maximal value corresponds to the situation when the worker's exclusive attributions are to supply materials; the same results are obtained if the worker will handle the machine exclusively, fact that points out the advantages of the specialization and of the collaboration in the production process.

Should we assume that the worker is handling both the machine adjustment and the supplying issues (to put it another way, $r_1 = r_2$), then we have:

$$y'_{opt} = -\frac{m_0 + xk + x}{r(k+1)} \quad (32)$$

To put it another way, in comparison to the situation pointed out by the formulas (31), only the $a \to 0$ requirement will be met, as the counter is independent with regard to the adjustment, and, in the case of the second situation, ($r_1 = r_2$), we will always obtain, in
comparison to the first, the following formula:

$$Y_{opt} < Y'_{opt} \quad (33)$$

Another fundamental value of the production process is the work productivity. Regarding this matter, managerial practice pointed out the fact that, as a larger number of identical products is executed, the execution duration per object diminishes (according to diagram 8).

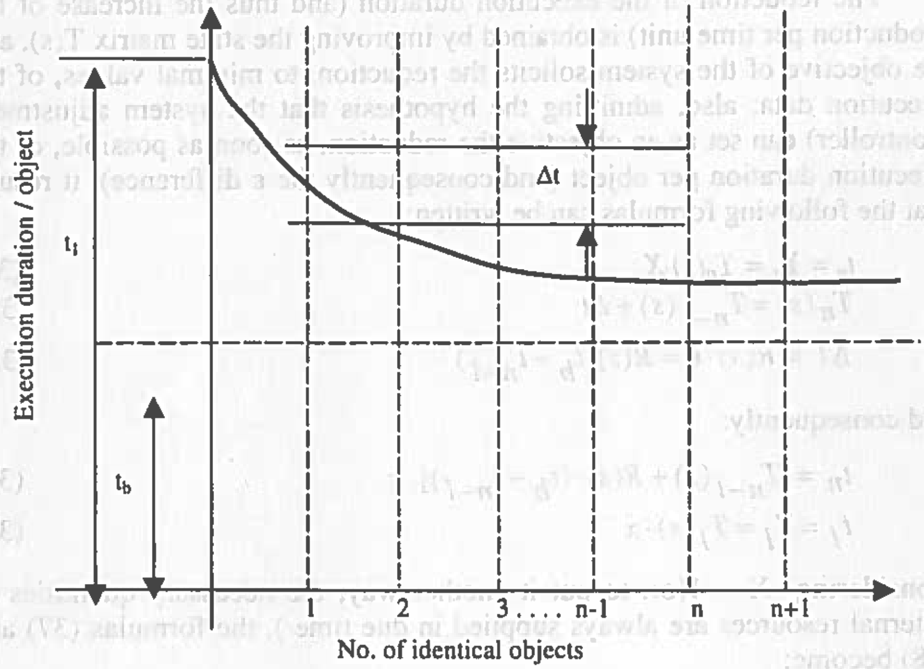

Fig. 8

Considering:

t_b = minimal duration of execution

Δt = the reduction duration of the time of execution when increasing their quantity

n = number of objects / products that are taken into account

t_1 = the execution duration of one (and only one) object.

Admitting that the system that executes more objects can be schematically represented as in figure 9, mentioning that the "reaction" of the regulator can be applied either to the entrances vector X, or to the state matrix T(s).

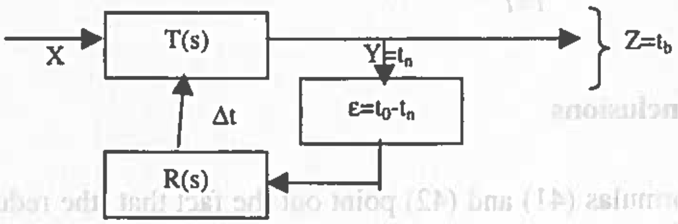

Fig. 9

The reduction of the execution duration (and thus the increase of the production per time unit) is obtained by improving the state matrix T(s), and the objective of the system solicits the reduction, to minimal values, of the execution data; also, admitting the hypothesis that the system adjustment (controller) can set as an objective the reduction, as soon as possible, of the execution duration per object (and consequently the ε difference), it results that the following formulas can be written:

$$t_n = Y_n = T_n(s) \cdot X \tag{34}$$

$$T_n(s) = T_{n-1}(s) + \Delta t \tag{35}$$

$$\Delta T = R(s) \cdot \varepsilon = R(s)(t_b - t_{n-1}) \tag{36}$$

and consequently:

$$t_n = [T_{n-1}(s) + R(s) \cdot (t_b - t_{n-1})] \cdot x \tag{37}$$

$$t_1 = Y_1 = T_1(s) \cdot x \tag{38}$$

Considering X = 1(or, to put it another way, the necessary quantities of external resources are always supplied in due time), the formulas (37) and (38) become:

$$t_n = T_{n-1}(s) + R(s) \cdot (t_b - t_{n-1}) \tag{39}$$

$$t_1 = T_1(s) \tag{40}$$

and the execution duration of the first object indirectly influences the initial state of the system, thus the (39) formula becoming:

$$t_n = T_1(s) + \sum_{i=1}^{n-1} R(s)(t_b - t_i) \tag{41}$$

or, in a simpler way:

$$t_n = t_1 + R(s) \sum_{i=1}^{n-1} (t_b - t_i) \tag{42}$$

3. Conclusions

The formulas (41) and (42) point out the fact that the reduction of the execution durations represent a direct consequence of the quality of the management system.

The estimation of the value of R(s) cannot be achieved considering the (42) formula only, as it contains another unknown quantity, that is t_b, that can be determined after the execution of enough identical objects (products); the formula is then:

$$t_b = t_1 + R(s) \sum_{i=1}^{N-1} (t_b - t_i) \tag{43}$$

where N designates the number of objects necessary in order to meet the t_n = tb requirement; developing the formula above, we will obtain:

$$t_b = t_1 + R(s) \sum_{i=1}^{N-1} t_b - R(s) \sum_{i=1}^{N-1} t_i \tag{44}$$

or:

$$t_b = t_1 + (N-1) t_b \cdot R(s) - R(s) \sum_{i=1}^{N-1} t_i \tag{45}$$

Of the last formula, we can extract either R(s):

$$R(s) = \frac{t_b - t_i}{(N-1) \cdot t_b - \sum_{i=1}^{N-1} t_i} \tag{46}$$

let t_b:

$$t_b = \frac{t_1 + R(s) \sum_{i=1}^{N-1} t_i}{1 - (N-1) \cdot R(s)} \tag{47}$$

In order to exemplify the formula above, let us suppose that, in a knit- wear goods factory, the head of a department wants to evaluate the efficiency of a foreman that must make 8 sports outfits during a shift, for which the execution durations (in events) were determined as corresponding to the values: 62, 55, 50, 48, 43, 38, 35 and 31; consequently, the value for the t_b was obtained through the execution of the 8 sports outfits(N = 8), t_b = 1 and t_1 = 62.

Applying the 46 formula, we have:

$$R(s) = \frac{31 - 62}{(8-1) \cdot 31 - 362} = \frac{31}{145} = 0.21$$

formula that indicates a foreman with very poor leadership qualities (generally for an R(s)∈[0.5; 0.67] we consider that the leadership qualities of the foreman are acceptable, for a value of R(s)∈[0.67;0.84] they are considered to be good, the leadership qualities corresponding to an R(s)∈[0.84; 0.93] being considered as very good.)

References

1. Elmagraby S.E., *Design of the production systems*, Ed. Tehnică, Bucharest, 1968.
2. Green H.W., *Econometric analysis*, EDI-MACMILLAN Publishing Company, 1993.
3. Malița M., Zidăroiu C., *Organization mathematics*, Ed. Tehnică, Bucharest, 1971.