ENTROPY OF STATE, FUNDAMENTAL PARAMETER TO THE DIAGNOSIS OF SOCIAL AND PRODUCTIVE SYSTEMS

1. Introduction

Let S be a system that at a certain point will be defined by an input a state, and an output vector.

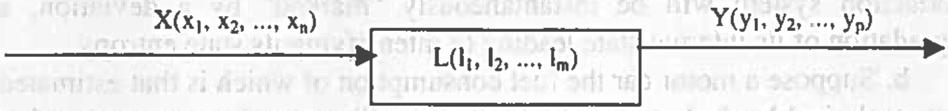

Fig. 1

We define the **internal state properly estimated** (denoted by L_0) as the state of the system when there is a complete correspondence between the control and response magnitudes (or in other words between the components of both the control and response vectors). Namely there is a control variable $x_0(x_1^0, x_2^0, ..., x_n^0)$ for which the response $y_0(y_1^0, y_2^0, ..., y_n^0)$ will be obtained if and only if the system state is as follows $L_0(l_1^0, l_2^0, ..., l_n^0)$.

The vector x_0 and the components $(x_1^0, x_2^0, ..., x_n^0)$ is called **the control vector in complete agreement with the state.** Consider the next hypothesis: to apply a control vector in complete agreement with the state to the system will not bring forth any response $Y \neq Y_0$.

Suppose that at one time the command $X \neq X_0$ is given a system and the response is $Y \neq Y_0$. In this case the system has an uncontrolable deviation, that is in some degree, its internal state suffered an instantaneous deterioration. We associate with this state of things the notion of state entropy and as we next

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shall see, it differs from the thermodynamic entropy although to take the variation speed of the last. Let us consider two examples: one from the social and productive area and the other from technics.

a. Let *A* be a production system the technical and material provisioning of which strictely complies with modern technology standards by both qualitative and quantitative points of view.

The other elements of production process are objectively considered as keeping to the standards. It is obvious that the system response (production regarded from the qualitative and quantitative points of view as well as costs) will be the one expected. Should a certain material be qualitatively inadequate, more expensive, or (quantitatively) insufficient, the system would register an exceeding of either specific consumption or costs. Also it may not achieve the production plan intended. Whatever the case, the production system will be instantaneously "marked" by a deviation, a degradation of its internal state leading to intensifying its state entropy.

b. Suppose a motor car the fuel consumption of which is that estimated in its technical book. Let us consider that it will respond to an acceleration command of 0-100 km/h after 12 seconds (presuming there is no defection). Should we use a fuel with the octanic number either superior or inferior to that in its technical book, the car would reach 100 km/h faster or, respectively, slower. During the few seconds the car will suffer an instantaneous deterioration of its internal state. The state entropy appears once again.

2. Estimation of state entropy

Next we shall attempt to elaborate a methodology to calculate the state entropy; it should be an estimation, a "measure" of a social and productive system's internal states and not an "indicator" (as it would have been impossible anyway: there is no device able to measure such a state and even if there had been one, it would have been still impossible because of the infinity of states that even a small and rather simple systemmay have).

We shall start by defining the number of states through which a system may pass. Between its 10 components (serial, parallel, or mixed) there are only 2 connexions that may occur. Graphically, such system appears like this:

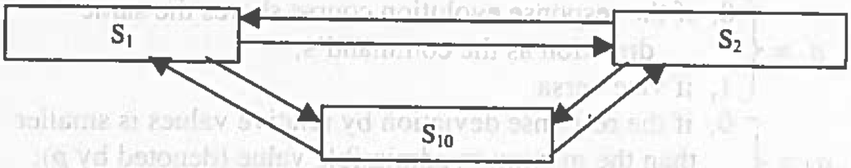

Fig. 2

Under the circumstances (small system with a small number of connexions between its components) the number of possible states will be the next:

$$2^{n(n-1)} = 2^{10(10-1)} = 2^{10\cdot 9} = 2^{90} \approx 1.3\cdot 10^{27}$$

It follows that for a small and relatively simple system the number of states is rather unconceivable. We will decrease the number to a more accessible one, especially needed by the practical point of view.

Hence we shall solve the following mathematic problem: time interval $[0,T]$ given, there are the next successive and discret time intervals $(t_k, t_{k+1}), k=\overline{1,n}$ (the fact that they are successive and discret does not encroach upon the generality of solution). On each interval commands x_k are applied to system A and the response magnitudes y_k are obtained. The state entropy of the system is to be estimated. To solve the problem one should know: **a.** the vector of state "a" and components a_1,a_2,a_3, is defined this way:

$$\|a\| = [a_1 \; a_2 \; a_3]$$

where: components a_i, $i=\overline{1,3}$ may have only the binary values 0 and 1;

b. the components of vector "a" are considered as follows:

- a_1 defines the correspondence between the evolution course of command and response directions;
- a_2 defines the correspondence between the response relative deviation as compared to the standardized, commanded response;
- a_3 defines the correspondence between the response absolute deviation as compared to the anticipated response.

c. the components of the state vector may have the next values:

$$a_1 = \begin{cases} 0, & \text{if the response evolution course shares the same direction as the command's;} \\ 1, & \text{if vice versa.} \end{cases}$$

$$a_2 = \begin{cases} 0, & \text{if the response deviation by relative values is smaller than the maximum admissible value (denoted by } p\text{);} \\ 1, & \text{if vice versa.} \end{cases}$$

$$a_3 = \begin{cases} 0, & \text{if the response deviation by absolute values is smaller than the maximum admissible value (denoted by } p\text{);} \\ 1, & \text{if vice versa.} \end{cases}$$

d. the state vector of complete correspondence is defined as:

$$\|a\| = [0 \;\; 0 \;\; 0]$$

From the above one should note that at the level of production system (regardless of size and nature) a standardized number of maximum 8 states has been obtained, according to the table below:

Table 1.

The Relationship between State and Its Decimal Correspondent

No.	State vector	The binary state of the state vector components	Binary complement	Decimal correspondent (C_Z)	Adjusted decimal correspondent (C_Z')
(0)	(1)	(2)	(3)	(4)	(5)
1.	a_0	0 0 0	1 1 1	7	8
2.	a_1	0 0 1	1 1 0	6	7
3.	a_2	0 1 0	1 0 1	5	6
4.	a_3	0 1 1	1 0 0	4	5
5.	a_4	1 0 0	0 1 1	3	4
6.	a_5	1 0 1	0 1 0	2	3
7.	a_6	1 1 0	0 0 1	1	2
8.	a_7	1 1 1	0 0 0	0	1

Now with every binary state its complement and decimal correspondent are associated. The data are to be found in columns (3) and (4).

In order to avoid the cases of undetermination when dealing with entropy calculus, we define the decimal adjusted correspondent (C_Z') according to the ratio:

$$C_Z' = C_Z + 1$$

Also we define the probability that at a given moment (t_k) the system had the state a_0 (command state, completely concordant) as the ratio between the correspondent adjusted to comply with vector a_0:

$$p_k = \frac{C'_{Z_{jk}}}{C'_{Z_0}}, \quad j = \overline{1.7}$$

Example: In three months a company intends to achieve the following:

1. to lower the costs by 2-5%;
2. the value of depress should amount to 1.3–2 thousand million lei;
3. to increase production by 1.5 - 2% (natural or natural and conventional units);
4. the additional value of production should amount to 2.5–3 thousand million lei.

The system state entropy as related to the targets is to be estimated knowing that:

1. cost depress went up to 1.57%;
2. depress value amounted to 1.2 thousand million lei;
3. production increased by 1.3% (natural or natural and conventional units);
4. additional value of production amounted to 2.8 thousand million lei.

There will be two calculus directions: the one of costs and the other of production.

As for the costs the comparative cases are like this:

- first point was partially achieved: a_1=1 and a_2=0;
- second point has not been achieved: a_3=0.

It arises that the state vector for costs has the components [*100*] and its adjusted decimal correspondent is 4 (tallying with a_4). The probability that may have a command tallying with the state is this:

$$p_1 = \frac{4}{8} = 0.50$$

For production the vector for components [*101*] (a_1=1, a_2=0, a_3=1), is obtained in the same way:

$$p_2 = \frac{5}{8} = 0.40$$

The state entropy has the formula below:

$$h = -\frac{\sum_{i=1}^{n} p_i \cdot \lg p_i}{n} \qquad (1)$$

where: n = number of factors reckoned with.

From our suppositional case (n=2) it arises that:

$$h = -\frac{0.5 \cdot \lg 0.5 + 0.4 \cdot \lg 0.4}{2} \approx -\frac{0.5 \cdot (-0.30103) + 0.4 \cdot (-0.39794)}{2} \approx 0.31$$

that indicates a low level of state entropy; moreover one may consider that:

- the state of production system is relatively good (the commands have been responded to);
- the decisional factor has not yet reached the qualitative level required.

3. Conclusions

Now we do have a calculus method of rather high certitude level able to indicates what exactly should and should not be expected from a production system.

Suppose that the maximum state entropy related to the system (h_{max}) is given:

$$R = \frac{h}{h_{\max}} \qquad (2)$$

Obviously, $0 \leq R \leq 1$. By means of this ratio the concept of production system **efficacy** is brought into discussion and given by the ratio below:

$$r = 1 - R \qquad (3)$$

Definition: **Efficacy** is that measure of the system functions referring to performance characteristics, namely its **capacity**, **safety** and **credibility** in responding in a **complete** and **aggregate** manner, without **unstructuring** processes to a **performant vector of command.**

In order to remove the possible confusions between the notions of efficacy and state entropy, we ought to give additional explanations.

From all the above one may infer that the state entropy appears instantaneously as a result of the "misunderstandings" between commands and the system state at a certain moment, misunderstandings that appear mainly and directly at the effecting structure by transferring it from the coordinates of a normal functioning (related to the system state) to the coordinates of either over- or under- functioning. These fluctuations between the relatively opposite or even extreme states strongly affects the system potential, leading (mainly) to a growing "distance" between the standard response and the real one.

If **the state entropy** is the measure for the instantaneous "degradations" caused by poor correspondence between commands and system state, the **efficacy** is the measure for establishing the number of "distances" (possibly increasing or decreasing); in other words, it is a "measure" for the system possibilities to be performant on Definition terms.

References

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